Teacher's Guide

Contemporary College Algebra: Data, Functions, Modeling

The two principal goals of the Contemporary College Algebra course are (1) to empower students to develop confidence in their problem-solving abilities in the modeling sense and (2) to empower students to become exploratory learners. As such, Contemporary College Algebra emphasizes student engagement, independent learning, critical thinking, and conceptual understanding. This is in contrast to a deep seated paradigm students have for how a mathematics course should be conducted—the instructor presents the material, works sample exercises, and then assigns similar problems for homework while the student memorizes the information and mimics the sample exercises. Thus the instructor's first, major challenge is to resolve this paradigm conflict in favor of the Contemporary College Algebra course.

Developing students to take responsibility for their own learning is an important component in the process of empowering students to become exploratory learners. The principal tactic in this development involves changing the paradigm of the role of homework from one of drill to one of discovery. For example, the typical homework assignment consists of students studying new material and working exercises over the new material before it is discussed in class.

Developing communication skills is an important objective of Contemporary College Algebra. In particular, strong emphasis is placed on how to learn from a textbook or instruction manual, how to make a class presentation (brief), and how to write. Most activities involve student presentations, while projects include both a class presentation and a written report.

Small group work is emphasized, starting with the first class. There are arguments pro and con concerning how groups are formed (instructor specified or student choice), how often group membership should be changed, and how group work is to be assessed. As the instructor, you should make decisions based on what seems feasible and workable for the class. Groups should be limited to a maximum of five students with three or four being preferable.

Provide students with guidelines for effective group work (e.g., involve each member, listen to each member, do not allow one person to do most of the work, be sure everyone understands the solution, take turns briefing results). You need to closely monitor the group work and provide frequent feedback (individual, group, class) on how well groups are functioning.

Reflection is a key component in learning and is central to personnel growth. Students need help in learning how to reflect, both with respect to the course material and to their own engagement with the course. You are encouraged to hold frequent reflection sessions during the semester. Peer feedback on group work can also be an important factor in student growth. One suggestion is to hold four or five peer feedback sessions during the semester in which each student fills in a simple peer review matrix (on his or her group

members) in which the column headings are the names of student's group members and the row headings are the following three questions:

a. Did he/she work hard on the group tasks?
b. How creative was his/her contribution to a group task?
c. Was he/she a team player (easy/hard to work with, reliable)?

The student fills in the matrix, rating each member on a scale of 1 to 5 (5 is best) for each question and then signs the peer review matrix. You can transform the responses into data and plot them (bar chart or scatter plot) to create a class picture of group engagement. Responses for an individual can be shared with the student during counseling sessions.

This Guide covers the sections of the text from Chapter 1 through Section 4.2. Some instructors may omit the matrix algebra in Section 2.8. (This would result in spending one day rather than two days on Section 2.8.) If Section 2.10 (linear Programming) is omitted, then Section 2.9 (Linear Inequalities) could also be omitted. All the other sections should be covered. Three hour tests (Lessons #12, 25, 37) and two Fun Projects are scheduled.

This Teacher's Guide is to be treated as a *Guide*. Instructors should feel free to modify the level and extent of the coverage as well as the pace while addressing the course goals and maintaining the spirit of the course.

Preparation (before classes start):

a. Make a poster listing the goals for the course (see the Preface) and post it in the classroom.
b. Make a poster showing the Problem-Solving/Modeling Process (Figure 1.1.2) and post it in the classroom.
c. Arrange chairs and desks into clusters for group work.
d. Prepare a Course Information handout including course goals; student responsibilities (e.g., doing homework, preparing for class, participating in class), participating in group work, attendance, grading schema, writing standards, class presentation standards, etc. (The handout is to be distributed to the class at the end of the first class period.)

Objectives for the first six classes:

a. Set the Tone for the course--student engagement, establish standards
b. Demonstrate how the course will be conducted—student participation, student responsibility (study new material and work exercises over it before coming to class, actively participate in small group work, develop communication skills).
c. Provide opportunities for students to experience success--homework, class participation, class briefings.
d. Create positive student attitudes.

Lesson #1 **Section:** Preface, Chapter 1

Objectives:

a. Students experience a small group activity that involves collecting data and conjecturing a model.
b. Learn how to plot a scatter plot by hand and with a graphing calculator.

Suggestive Lesson Plan: (Course information is not discussed until the second class.)
Group Activity: Spend 30-40 minutes on the Handshake Activity

a. Divide the class into groups (3-5/group).
b. Ask the group members to introduce themselves to each other.
c. Present the Handshake Problem: A group of six people meet and each person shakes the hand of every other person exactly once. Task each group to determine the total number of handshakes for the group of six people.
d. Walk around the room observing how the groups are approaching the problem. (It is alright if the students are just "floundering around.") If one (or more) of the groups is pursuing a promising approach, ask them to share their reasoning with the rest of the class. In particular, if a group is creating a chart representing the number of handshakes, ask them to write their chart on the board and explain their reasoning to the rest of the class. If there are some groups that seem stymied, suggest their making a two-column table with one column, say the left column, labeled "Number of People" and the other column labeled "Number of Handshakes." After a few minutes, ask one of the students to write their table on the board and explain how they determined the number of handshakes.
e. Ask the class if anyone has a different approach? Encourage students to describe different methods.
f. Pose the question: How could you extend your method (e.g., table) to include eight people? *n* people? Have students present their answers to the class.

(Tell the students to keep a record of their data as it will be used later in the course in curve fitting and again in modeling with recursive sequences.)

Use the handshake data to illustrate how to sketch a scatter plot by hand and then by using a calculator. Pass out the following instruction sheet on how to how to create a scatter plot using a calculator.

Pass out the Course Information material including course goals; student responsibilities (e.g., doing homework, preparing for class, participating in class), participating in group work, attendance, grading schema, writing standards, class presentation standards, etc.

Homework:

Read and reread the Course Information material.
Read and reread the Preface and Chapter 1 of the text.
Write a one page essay (to hand-in for grade) summarizing aspects of the Course Information, the Preface, and Chapter 1.

Instruction sheet for forming a scatter plot of a two-column table of data using a TI 83 graphing calculator:

- Turn on and press the **STAT** key.
- Move the cursor to **Edit** and press **ENTER** (columns labeled L1, L2, L3, … should appear in the window).
- Clear columns L1 and L2--Use the up key to move the curser to a column label, say L1, press the **CLEAR** key, and then press **ENTER**. Repeat for column L2.
- Enter the data in the left column of the table into column labeled L1--type an entry and then press **ENTER**.
- Enter the data in the right column of the table into column labeled L2—type an entry and then press **ENTER**. (Check that columns L1 and L2 have the same number of entries.)
- Press **2nd** and **STAT PLOT** keys.
- Press **ENTER** and move the blinking curser to **On** and press **ENTER**.
- Select the type of graph--move the curser to the scatter plot picture following Type: and press **ENTER**.
- Select the type of **Mark** you want—move the curser to the mark and press **ENTER**.
- Set the viewing window dimensions—Press **WINDOW** and make the appropriate choices.
- Press the **GRAPH** key.

A line plot is obtained by changing the eighth bullet (select the type of graph) so that the curser is on the picture of the line graph and then pressing **Enter**.

Reference: TI-83 Graphing Calculator Guidebook section 12-31

Lesson #2 Section: Preface, Chapter 1

Objectives:

a. Students make class presentations (reading their essays).
b. Students understand the Course Information and what is expected of them.

Suggested Lesson Plan:

Activity: Ask several students to read their essays to the class. Use the essays to generate a class discussion. (If necessary, instructor poses questions.)
Some sample questions:

- What does it mean to you to be an exploratory learner?
- What does it mean to learn how to learn? To be an independent learner?
- What responsibilities do you have to make the course successful?
- How important are effective communication skills to you now and in the future?
- How do you develop effective communication skills?
- How do you develop self-confidence as a problem solver?
- What does it mean to mathematically model a real-life situation?
- What does the term (mathematical) function mean to you?
- What are some examples of the "conjecture and test" method that you have experienced outside of mathematics (e.g., practicing a sport)?
- What does the term (mathematical) variable mean?

Call the class's attention to how the Communication Goal has been addressed in the first two classes--reading (Preface, Chapter 1), writing (essay), presenting (class), discussion.

Small Group Activity: Create a two-column table with the left column listing the first ten positive integers and the right column listing the accumulated sums (e.g., the fourth entry is 1+2+3+4=10); and then form a scatter plot of the data in the table. Ask a student to sketch his/her scatter plot on the board and then describe properties of the plot (e.g., increasing to the right, tends upward, rate of increase increases).

Ask for volunteers to show the class how to plot a function using the calculator. Then have everyone plot $y = x^2$, $y = 2^x$.

If time permits, have everyone plot $y = e^{-x}$ *and* $y = \frac{1}{x}$ on the same set of axes (multigraph) using a window of $-5 \le x \le 5, -5 \le y \le 5$. Then ask someone to sketch their graphs on the board and explain how they can distinguish between the graphs.

Suggested Homework:

Study Section 2.1 (Displaying Data);
Work Exercises: 2.1/1, 2, 5, 7, 15

Lesson #3 Section 2.1 (Displaying Data)
Need a copy of a *USA Today* newspaper for each group

Objectives:

a. Students understand how to display data using tables, bar charts, line plots, and scatter plots.
b. Students understand how to compute percentages.

Suggestive Lesson Plan:

"Wake up activity" (5-10 minutes): Put three percentage problems (similar to Exercises 2.1/1, 2) on the board for everyone to work. Ask some of the students to explain their work and their reasoning. The purpose is to "wake up the minds" and engage students.

Comment on how to make a brief (presentation) to the class. Some examples are:

a. Stand straight.
b. Make eye contact.
c. Speak clearly.
d. Be respectful of instructor and other students.
e. Write only the necessary details on the board (if everyone has a copy of the problem, it is not necessary to write the problem on the board).
f. Be prepared to elaborate and provide any missing steps.
g. Do not block board work by standing in front of it.
h. Ask if everyone understands and is following the argument.
i. Ask for questions or comments.
j. Show pride in your work.

You need to continually establish and enforce a "standard" for making a class brief.

Small Group Activity (20-25 minutes): Exercise 2.1/9 (you supply newspapers). Call on different groups to show their examples to the rest of the class, explaining why the particular method selected for displaying the data was appropriate. What is being communicated? Interpret the editor's intent.

Bring out the points:

a. A line plot should not be used when the underlying setting is discrete (it does not make sense to interpret between data points in a discrete setting)
b. The area of a pie slice in a pie chart represents the entry's percentage of the total amount.

10 minutes: Ask a student to put homework Exercise #15d on the board
and explain the reasoning involved. Then invite student comments.

Ask for questions on the homework (deflect computational questions to other students).

Call on students to answer Queries #1, 2.

Suggested Homework: Study Section 2.2; Answer Query # 2
Work Exercises 2.2/2, 5, 14, 17, 18;

Lesson #4 Section 2.2 (Average (Mean))

Objectives:

a. Students understand definition of average (mean) and weighted average.
b. Students understand how to compute both an average and a weighted average. (Examples of a weighted average: grade point average, semester test average (final exam weighs more than a regular test))

Suggested Lesson Plan:

Small Group Activity: Assign each group one of the homework exercises to put on the board and to explain to the class how they obtained their solution. Each group concludes its presentation with What-ifing their exercise by making up two similar examples and working one of them.

Small Group Activity: (difficult activity, reserve for a strong class) Ask students to read each of the following two questions, but not to write anything until you tell them to. Then

a. Ask students to estimate the answers. Call on one or two students to explain the basis for their estimates.
b. Ask students to draw a picture showing what is given and what the question is asking.
c. Ask students to identify what they need to know in order to answer the question. (For example, they need to know the time for the trip in Part a and the time for the last 30 miles in Part b.)
d. Ask someone to define rate (rate = $\frac{\text{distance (miles)}}{\text{time (hours)}}$) and then show how to determine time in terms of rate and distance.
e. Allow students, working in groups, to write their computations in order to answer the questions. Remind them to define their variables.

Questions:

1. On a 60-mile car trip, you average 40 mph over the first 30 miles and 60 mph over the last 30 miles. What is your average speed for the entire trip?

2. On a 60-mile car trip, you average 40 mph over the first 30 miles and 50 mph over the entire trip. What is your average speed over the last 30 miles?

Call on different groups to explain to the class what they did and their reasoning.

Ask the students to explain why their answers are reasonable.

Ask for questions on the homework (direct computational questions to other students).

If time permits, Lead a discussion on the use of averages (e.g., reduces errors with data, summarize data, etc.)

Suggested Homework: Study Section 2.3 (Median & Mode);
Work Exercises: 2.3/2, 8, 9, 10, 19, 22

Lesson #5 Section 2.3 (Median and Mode)

Objectives:

a. Students understand the definitions of median and mode for a numerical data set and how to compute them.
b. Students contrast the three measures of central tendency: average, median, mode.
c. Students experience different ways of solving a problem.
d. Students experience working with integers.

Suggested Lesson Plan:

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises.

Small Group Activity: (Instructor directed)

a. Ask for an example of 10 positive integers having an average of 8. Call on students to explain how they obtained their example. (There will probably be two or three or more different methods, including 10 eights; "balancing" by inserting pairs of integers that have an average of 8 (e.g., 7 and 9, 6 and 10); recognizing that the sum must be 80 and thus any set of positive integers summing to 80 is acceptable)

b. What-if the problem, asking for an example of 10 positive integers having an average of 8 and a median of 5. Call on students to present their example and their reasoning.

c. What-if the problem, asking for an example of 10 positive integers having an average of 8, a median of 5, and a mode of 3. Call on students to present their example and their reasoning.

If time permits, use Exercise 2.3/16 for a group activity leading into a class discussion. (Do not attempt to form a rigorous definition of *outlier*. Make it clear that a person should consider the context of a data value before eliminating it as an outlier.)

If time permits, initiate a discussion on the use of average and median. For example in a debate on a proposed tax cut, should one cite the average projected savings or the median projected savings?

Suggested Homework: Study Section 2.4 (Variable Representation);
Work Exercises: 2.4/1, 2, 4, 7, 13

Announce a Quiz for the next class

Lesson #6 Section 2.3 (Variable Representation)

Objectives:

a. Understand the definition of variable.
b. Understand how variables are essential to problem solving.
c. Understand the roles of dependent and independent variable in a relation.

Suggested Lesson Plan:
Quiz (e.g., 10-15 minutes, three questions) Suggested questions:

(1) Display the given data in a bar chart. (Copy a data set from *USA Today*. Include a segment of graph paper on the quiz paper.);
(2) Gretchen's (hour) test scores this semester have been 72, 77, 81, and 86. If the weight of the Final Exam is twice the weight of an hour test, what score would she need to get on the Final Exam to have an 80 average for the course?
(3) Suppose the median height is less than the average height for a group of nine individuals. Determine if there are more people in the group that are taller than the average or more that are shorter than the average. Explain your reasoning.

Small Group Activity: Assign an exercise to each group (e.g., 2.4/10, 11, 12, 14, 15) to work and then brief to the class. Emphasize setting up the problem, in particular defining the variable. For the remainder of the course, you will need to stress the importance of defining variables.

As a follow-up to the Small Group Activity, lead a class discussion on the use of an appropriate symbol (e.g., letter) to denote a variable.

Ask for questions on the homework (direct computational questions to other students).

Assign Fun Project2.1: Poster Project. Due on Lesson #10
(Provide students guidance on making their poster: e.g., size of the poster, use of poster board, all charts and graphs should be labeled, written paragraphs should be typed. Suggest that students look at other posters for display ideas.) Tell the students that their posters will be judged by a team of faculty members.

Ask your Department Chairperson and one or two other faculty members to view the finished posters and decide on one that would be displayed on a departmental bulletin board. (This would add a bit of competition, spread word of your program, and provide an opportunity to showcase student work.)

Suggested Homework: Study Section 2.5 (Circle Properties and Pie Charts)

Work Exercises: 2.5/1, 2, 10, 11
Work on the Fun Poster project.

Arrange for someone in the computer lab to show the students how to form pie charts using Excel.

Lesson #7 Section 2.5 (Circle Properties and Pie Charts)

Objectives:

a. Understand how to display data in a pie chart. In particular, emphasize that the size of each category represents a percentage of the whole.
b. Understand that the terms percentage, proportion, and relative frequency all mean the same thing and are used interchangeably.
c. Understand how to convert between degree and radian measure.
d. Understand circle properties: area (πr^2), circumference ($2\pi r$), total central angle (2π), proportional reasoning to obtain arc length and area of a pie slice.

Suggested Lesson Plan:
Return quiz (create opportunities to praise and make positive comments).

Reflect over the past six lessons. Be as specific as possible, starting off with positive comments and then addressing corrective measures that need to be taken. Draw attention to

a. How the work addressed the goals of the course (point out the classroom poster listing the course goals).
b. Skill improvement in working with numbers, percentages, and solving linear equations.
c. Improvement in communication (e.g., class presentations).
d. Improvement in group work and general class rapport.
e. Development if independent learning (e.g., homework)

Encourage students to make and express their own reflections.

Small Group Activity: Write five exercises on the board similar to the exercises in 2.5/1, 2 for the students to work. After approximately six minutes, call on individual students to present their work (one for each exercise) and then to what-if their exercise by making up and working a similar one.

Small Group Activity: Each group works Section 2.5/18 (update the cost of gasoline), then approximates a "fair" increase in price for gasoline if the price of a barrel of crude oil increased by $4.00. Require each group to list their assumptions. (A barrel of crude oil contains 42 gallons.)

Ask one group to brief the class on their work and then have that group lead a class discussion on the different assumptions and approaches used by the other groups. (Emphasize the need and importance in making assumptions when working with real-life situations.)

Ask students what they learned from their homework? Ask for questions on the homework, particularly on the proportional reasoning used in determining area of a pie slice and arc length.

Suggested Homework: Study Section 2.6 (Discovering Relations Between Variables)
Work Exercises 2.6/2, 4, 5, 7, 11
Work on the Fun Poster Project

Lesson #8 Section 2.6 (Discovering Relations Between Variables)
Two class periods should be allocated to Section 2.6.

Objectives:

a. Understand how to plot a (linear) relation.
b. Understand the meanings and the roles of independent and dependent variable.
c. Understand the meaning and role of slope and intercept.
d. Understand how to determine the equation of a line.

Suggested Lesson Plan

Small Group Activity: Assign each group one of the homework exercises to put on the board and to explain to the class how they obtained their solution. Each group concludes its presentation by explaining why they think their particular problem had been assigned for homework.

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises. Ask for questions on the homework (direct computational questions to another student).

Skill work--Present a problem, everyone works the problem, and then the instructor calls on a student to brief their solution and reasoning. Example problems:

a. Determine the equation of the line passing through the points (1, 2) and (4, -2).
b. Determine the equation of the line passing through the point (2, 3) with slope 4.
c. Determine the x and y intercepts of the line $2x - 4y = 6$.
d. Answer Query 7 in Section 2.6.
e. Answer Query 10 in Section 2.6.

Suggested Homework: Study Section 2.6 (again)
Work Exercises 2.6/1, 10, 12, 14, 17, 19
Work on the Fun Poster project

Announce a quiz for next class. Comment that the grade on each problem is dependent on how the problem is set up (e.g., defining variables) as well as the computations

Lesson #9 Section 2.6 (Discovering Relations Between Variables)
Second of two class periods allocated to Section 2.5.

Objectives:

a. Understand the meaning and role of slope and intercept.
b. Understand how to determine the equation of a line.
c. Students experience a conjecture and test process leading to a solution.

Suggested Lesson Plan
Quiz #2 (10-15 minutes) Suggested problems

a. Convert $\frac{3}{4}\pi$ to degree measure.
b. Determine the value of b that satisfies the equation $3(x-b) = 4x+1$ when $x = 2$.
c. Determine the dimensions of a wall poster whose area is 50 square feet and whose width is twice as long as the height.

Class Activity: Modify the "Line of Sight" activity in Section 2.6 to a class activity. Project the graph of the skyline on the board and then ask a student to conjecture the equation of the desired line; project the graph of the skyline and the conjectured equation (compliment the student for the correct portion such as having the correct numerical sign for the slope); call on another student to describe how the equation should be changed to obtain a better approximation; change the equation as suggested and project the new graph; ask another student how the new equation should be changed to obtain a better approximation; change the equation and project the new graph; continue this process until a suitable approximation is found. (List the different approximations on the board.) (If projection facilities are available, this activity could be carried out using the text's CD-Section 2.6.)

Compliment the class on how well they carried out the conjecture and test process based on their understanding of slope and vertical intercept. The willingness to make an attempt (conjecture), note the errors (test), and then make an improved attempt is central to becoming a successful problem-solver.

Follow-on Small Group Activity: Give each group a similar problem to work (i.e., give each group a fourth degree equation and ask them to determine the equation of a "line of sight"). Section 2.6 of the CD has several examples of the "Line of Sight" problem.

Ask students what they learned from the homework? Ask for questions on the homework (direct computational questions to another student).

Be prepared to instruct on how to work with fractions. Some skill work on manipulating simple fractions may be needed.

Suggested Homework: Study Section 2.7 (Applications of Linear Equations)
Work Exercises 2.7/1, 2, 4

Complete the Fun Poster project

Lesson #10 Section 2.7 (Applications of Linear Equations)
Two class periods should be allotted to Section 2.7
Objectives:

a. Understand the form of a linear equation.
b. Understand how to solve a linear equation symbolically and graphically.

Suggested Lesson Plan
Collect Fun Posters. As you collect the posters, look for particular aspects on which to comment (be positive). Initiate an informal class discussion on the Fun Poster project. Suggested questions:

- What was the most difficult part of doing the project?
- Did you learn anything by doing the project? What?
- How well did your group work together?
- Reflecting on your experience, can you identify aspects of the group process that could be improved upon?
- Are you proud of your work?

Conduct Peer Review #1 (5 minutes, students fill in peer review matrix)

Tell the students which faculty members will be judging the posters.

Return Quiz #2.

Reflect on the growth you have seen, be upbeat, and encouraging. Point out how the quiz questions addressed lesson objectives/course goals. Ask the students to comment on how they feel about their learning, their attitude, their ability to work together on group activities.

Ask for questions on the homework (deflect computational questions to other students). Ask for questions on past work. This is a good time for "catch up" work if needed.

Small Group Activity: Assign each group a problem to work and then to brief to the class. Suggested problems: 2.7/5, 9, 10, 11, 12

Suggested Homework: Study Section 2.7 (Applications of Linear Equations)
Work Exercises 2.6/13, 16
Work Exercises 2.7/15, 16, 17, 21

Lesson #11 Section 2.7 (Applications of Linear Equations)
Second class over Section 2.7

Objectives:

a. Understand the form of a linear equation.
b. Understand how to solve a linear equation symbolically and graphically.

Suggested Lesson Plan
Assign each of the homework exercises to a student to brief their solution to the class.

Ask a student to explain how to solve an equation graphically. After the explanation, give the student an equation to solve graphically illustrating his/her explanation.

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises. Ask for questions on the homework (deflect computational questions to other students).

Small Group Activities: Section 2.7/5, 16, 25

If time permits, conduct an additional Small Group Activity: Section 2.7/6, 7, 8

Announce a test. Lead a discussion on how to review for the upcoming test.
(Ask each student if he/she has met each of the lesson objectives.)

Suggested Homework: Review for test

Lesson #12 Sections 2.1-2.7 (Review)

Objective:

Review for test over Sections 2.1-2.7.

Suggested Lesson Plan
Ask students what they did for review.
Respond to questions from students' review
Ask individual students to illustrate/comment on each of the Objectives in Lessons #1-9

As time permits, students work problems from the following list:
2.1/11; 2.2/4; 2.3/11; 2.4/10; 2.5/6; 2.6/6; 2.7/21

Suggested Homework: Prepare for the test.

Lesson #13 Test

Suggested Questions: (select five questions including at least two of the last three)

a. Display given data as a scatter plot (graph paper provided)
b. Display given data in a table and in a line plot (graph paper provided).
c. Given a pie chart, compute the percentage for a specified entry.
d. Determine a set of six positive integers that has an average of eight and a median of five.
e. Determine the equation of a line with slope 3 that has an x-intercept of 5.
f. Determine a linear equation that represents the set of points whose distance from the y-axis is four times its distance from the x-axis.
g. Solve the equation for x: $3x + 7(2-5x) = 4x$.
h. Jane paid the wholesale price of $50 for a dress and then marked it up 30% for sale in her dress shop. The dress did not sell and she later reduced the store price by 30%. Determine the final sale price for the dress and compare it to the original wholesale price.
i. Alex bought desserts for ten people, including himself. Three of the desserts cost $3 apiece, three desserts cost $2 apiece, and four desserts cost $1 apiece. He told his friends, "The desserts cost $1, $2, and $3. So the average cost is (1+2+3)/3=$2. If each of you gives me $2, we will be even." Did Alex gain, lose, or break even? Explain your reasoning.
j. A vender sells hot dogs at a price of $1.75 each and chips at $.50 a bag. If he sold 20 more hotdogs than bags of chips and his total revenue was $192.50, how many hotdogs did he sell? Show how you set up the problem along with your solution.

Suggested Homework: Study Section 2.8 through page 73 (Systems of Equations)
Work Exercises 2.8/1 (no matrix method), 4, 5, 6, 7

Lesson #14 Section 2.8 (Systems of Equations)
Plan to spend two lessons on this Section.

Objectives:

a. Understand, symbolically and graphically, what it means to solve a system of linear equations.
b. Understand the graphical, substitution, and elimination methods for solving a system of equations.

Suggested Lesson Plan
Ask a student to explain to the class what it means to solve a system of equations.

Assign each homework problem to a student with instructions to write their solution on the board and then brief it to the class. (If a student was not able to solve a problem, have the person put their attempt on the board and then let the class finish the problem.)

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises. Ask for questions on the homework (direct computational questions to other students).

Ask a student to brief the class on the elimination method for Example 2.8.3, explaining each of the steps on page 72.

Small Group Activity: 2.8/3.
After a suitable time, call on a student to brief the class on 3a. Then call on another student to brief the class on 3b. Expand this briefing into a discussion on solving a system when there are:

a. The same number of variables as unknowns (e.g., 3a)
b. Fewer variables than unknowns (e.g., 3b)
c. More equations than variables.

Challenge the class to make up an example of a system of three equations in two variables that does not have a solution. Verify that the system has no solution by having a student graph the three equations on the board.

Challenge the class to make up an example of a system of three equations in two variables that has a unique solution. Verify that the system has a solution by having a student graph the three equations on the board.

Suggested Homework: Study Section 2.8 beginning on page 74 with the Matrix method.
Work Exercises: 2.8/9, 10, 12, 13, 14
Consult the graphing calculator instruction manual for how to solve a system of equations using the rref command.

Instructor's note: If you prefer not to include matrix algebra in your course, revise the next lesson to include (1) return and analysis of Test 1, (2) Reflection based on the results of Test 1, and (3) additional work on systems of equations.

Lesson #15 Section 2.8 (Systems of Equations)
Second lesson on this Section.

Objectives:

a. Return Test #1 and reflect on Lessons 1-13
a. Understand definitions of
    Matrix
    Row equivalent
    Row operations
b. Understand the matrix method for solving a system of equations.

Suggested Lesson Plan

Return Test #1 – Share your analysis of the test results with the students (be positive, look for aspects beyond the grade on which you can compliment the students).

Reflection: Engage the students in reflecting on Lessons 1-13:
What went well (effective, useful, challenging, worthwhile, etc.) and what didn't;
Level of understanding;
Success in addressing the goals of the course;
Group work (effectiveness, everyone sharing, learning activity);
"Student Growth" in learning how to learn (e.g., study habits);
Level of self confidence;
Preparation for Test #1;
Change in attitude;
Etc.

Assign each homework problem to a student with instructions to write their solution on the board and then brief it to the class. (If a student was not able to solve a problem, have the person put their attempt on the board and then let the class finish the problem.)

Initiate a discussion (based on homework questions) on the homework.

Have the class answer each of the Queries in Section 2.8 and then call on particular students to explain their reasoning.

Small-Group Activity: Assign each group a chemical equation to balance using matrix algebra (select equations from Exercises 16-18). Have the groups brief their work.

If time permits, have the groups work 2.8/26

Suggested Homework: Study Section 2.9 (Linear Inequalities)
Work Exercise 2.8/21
Work Exercises 2.9/1(a,b), 2(a,b), 4(a,b), 5(b,c), 7, 17

Instructor's note: If you prefer not to include Linear Programming in your course and you are pressed for time, you may skip the next two lessons.

Lesson #16 Section 2.9 (Linear Inequalities)

Objectives:

a. Understand the meaning of
   $x < 8$
   $x + 4 < 3x - 6$
   $|x - 2| < 4$
b. Understand how to solve an inequality expression graphically and symbolically.
c. Understand the definition of absolute value as denoting distance.
d. Understand how to express intervals using inequalities and absolute values.
e. Understand that a linear inequality defines a half-plane.

Suggested Lesson Plan
(Do not let the class get "bogged down" attempting to solve complicated absolute value-inequality exercises. The skill level that is necessary is that for solving linear inequalities and expressing intervals using absolute value-inequality notation (e.g., Objective a).)

Ask a student to brief the class on their work on 2.8/21.

Small-Group Activity: Assign Exercise 2.8/22. Compare the answers from the different groups.

Assign each of the homework exercises on inequalities to students to work on the board and then explain their reasoning to the class.

Have all the students work 2.9/17 and then discuss the results. In particular, discuss how a linear inequality defines a half-plane. (Be sure that students understand that a linear inequality in two variables defines a half-plane and how to determine that half-plane.)

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises.

Ask for questions on Sections 2.8 and 2.9 (direct computational questions to other students).

Suggested Homework: Study Section 2.10 (Linear Programming)
Work Exercises 2.10/1, 2, 5, 6

Lesson #17 Section 2.10 (Linear Programming)

Objectives:

a. Understand the definition of Linear Programming.
b. Understand the meaning of the terms:
   Objective Function
   Constraint equations
   Feasible Region
c. How to graphically solve a linear programming problem in two variables.

Suggested Lesson Plan
(Consult a faculty member in the School of Business for background information (e.g., how is linear programming presented and used in the School of Business). Invite the faculty member to visit your class and talk to the students about the role of linear programming in business.)

Discuss the corn/wheat problem in Section 2.10. In particular, call attention to how linear programming is used to show how the government can influence planting strategies through crop subsidies.

Ask for questions on the homework (direct computational questions to other students).

Small-Group Activity: Work exercises 2.10/8, 11, 12 as time permits. Insist that students define their variables and their objective function.

The purpose of studying Section 2.10 is to illustrate a problem-solving technique that is important in business and serves as a link between college algebra and the School of Business. The corn/wheat problem illustrates the use of linear programming in establishing the government's farm subsidy policy.

Suggested Homework: Study Section 3.1 (Functions)
Work Exercises 3.1/1, 2, 4, 14

Announce a quiz for next class. Comment that the grade on each problem is dependent on how the problem is set up (e.g., defining variables, model) as well as the computations

Lesson #18 Section 3.1 (Functions)

Objectives:

a. Understand the graphic, symbolic, and numeric methods for representing functions.
b. Understand and visualize a function as an input-output process.
c. Understand how to evaluate a function.
d. Understand the meaning of a zero of a function.

Suggested Lesson Plan

Quiz #3 (10-15 minutes) Suggested Problems:

a. Cora's bill for a new dress is $60.99 including the 7% sales tax. What was the price of the dress before the tax was added?
b. Willie paid $3.90 for 2 sodas and 4 bags of chips and Julie paid $1.50 for one soda and two bags of chips. Determine the cost of a soda and of a bag of chips.
c. Determine the values of x that satisfy the inequality $3(2 - x) > 4x - 6$.

Ask students to talk about what the word "function" means to them mathematically. Urge the students to expand their discussion to include the pros and cons of the graphic, symbolic, and numeric representations of functions.

Ask for questions on the homework (direct computational questions to other students).

Work Exercises 5-12 as a class activity. Give everyone 10-15 minutes to work the exercises and then begin calling on different students to brief an exercise to the class. Insist that the student verbally justify the shape of the graph he or she drew.

As time permits, write several functions on the board (similar to those in #4) and then ask the students to graphically approximate the zeros.

Ask the class how they can tell graphically if a zero is even (occurs an even number of times) or odd (occurs an odd number of times). If the value of a function is positive just to the left of an even zero, is the value of the function positive or negative just to the right of the zero? Explain.

Suggested Homework: Study Section 3.2 (Definitions)
Work Exercises 3.2/1, 2, 4, 25, 26

Lesson #19 Section 3.2 (Definitions)

Objectives:

a. Understand the definitions of all the bold-faced terms in Section 3.2.
b. Understand the vertical line test.
c. Understand functional notation.

Suggested Lesson Plan

Return Quiz #3 Comment on how well the students are setting up their problems.

Check on the understanding of the bold-faced terms in Section 3.2.

Check on the understanding of the vertical line test. In particular, ask if it is true that given a function, every vertical line must intersect the graph of the function exactly once?

Check on the understanding of function notation. (For example, $f(2)$ is the output value corresponding to the input 2.)

Conduct a class Example and Verification Activity. Present a scenario to the class and allow the students a minute or two to think of an example. Then call on a pair of students, asking one to give an example of the scenario and the other to verify the correctness of the example. Continue this process until every student has been engaged. Some suggested scenarios are:

A relation that is not a function.
A relation that is a function.
A function with domain {2, -3, -4, 5, 7} and range {2, 3, 4}.
An increasing function.
A relation with domain {1, 3, -4, 5, 6} and range {5, 7, -2, 6} that is not a function.
A function whose zeros are 0, 2, and 4.
A function that has a double zero at $x = 2$.
A function whose domain contains all real numbers and whose range is [-1, 1].
A function whose domain is $\{x \mid x \geq 2\}$.
A function whose range has more elements than the domain.
A relation that is a function, yet the vertical line $x = -2$ does not intersect the graph of the relation.

Small-Group Activity: Assign each group to work Exercise 3.2/28. After an appropriate time, ask one of the students to brief the class on the exercise.

As time permits, call on individual students to work Exercises 3.2/5-11, 13-25.

Suggested Homework: Study Section 3.3 (Predictions Based on Data)
Work Exercises: 3.3/2, 4, 5, 17, 19

Lesson #20 Section 3.3 (Predictions Based on Data)
Two classes should be allocated for Section 3.3.

Objectives:

a. Understand the five basic functions.
b. Understand and be able to recognize the shapes of the graphs of the five basic functions.

Suggested Lesson Plan
Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises.

Ask for questions on the homework (direct computational questions to other students).

Small Group Activity (See Exercises 6-10): Assign a function from the following list to each group. Ask each group to develop and present to the class a discussion/demonstration on the effects of changing the *c* or *n* in the description of their function. The demonstrations should include both graphical illustrations and algebraic reasoning.

Functions: $f(x) = cx^2$; $f(x) = cx^3$; $f(x) = x^n$; $f(x) = \sqrt{cx}$; $f(x) = x^{\frac{1}{n}}$

Small Group Activity (See Exercises 11-15): Repeat the previous activity with the following functions:

Functions: $f(x) = c\,10^x$; $f(x) = \log(cx)$; $f(x) = c\sin(x)$;
$f(x) = c\cos(x)$, $f(x) = \cos(cx)$

Small Group Activity (discovery): Without calculating, sketch and label the graphs of each of the following functions on the same set of axes.

Functions: $f(x) = 10(0.5^x)$, $g(x) = 10(0.2^x)$, $g(x) = 10(1.2^x)$

Repeat with another set of exponential functions that include bases less than one and greater than one. Make a conjecture about the shape of the graph of an exponential function in reference to whether the base is less than one or greater than one. Justify your conjecture algebraically. Share your reasoning with the rest of the class.

Conduct Peer Review #2 (5 minutes, students fill in peer review matrix)

Suggested Homework: Study Section 3.3 (Predictions Based on Data)
Work Exercises 3.3/16, 18, 20, 21 (Hint: Show that both sides of the conjectured equality are equal to the same thing and thus equal to each other. Because the logarithmic and exponential

functions are inverses of each other, $\log_b(b^a) = a$ and $b^{\log_b(a)} = a$. Thus $b^{c\log_b(a)} = a^c$ and therefore $c\log_b(a) = \log_b(a^c)$. Explain the last step.

Lesson #21 Section 3.3 (Predictions Based on Data)
Second class on Section 3.3.

Objectives:

a. Understand that the exponential and logarithmic functions are inverses of each other.
b. Understand and be able to identify the shapes of the graphs of the five basic functions.

Suggested Lesson Plan

Quiz #4 (10-15 minutes) Suggested Problems:

a. (i) Give an example of a function whose domain is {2, -3, 1, 5, -2} and whose range is {3, 5, 8}.
b. (ii) Give an example of a relation that is not a function and whose domain is {2, -3, 1, 5, -2} and whose range is {3, 5, 8}.
c. Include the graph of a piecewise, non-continuous curve on the quiz paper and ask: Is the curve the graph of a function? Explain your reasoning.
d. Identification problem similar to Exercise 3.3/16 or 17.

Questions on homework (direct computational questions to other students).

Discuss exponential functions—help the class identify several scenarios involving exponential functions (unconstrained growth of bacteria, radio active decay (strontium-90), compound interest, musical pitch, number of a person's ancestors, number of people in a phone tree, decay rate of chlorine in a swimming pool, etc.).

Check to make sure that everyone in the class understands the savings account illustration in the text of an exponential function.

Lead a class discussion on a comparison of linear, quadratic, and exponential functions. In particular note that a linear function represents (models) a quantity whose rate of change is a constant (e.g., a house painter's wage is a linear function of the number of hours worked); and that an exponential function represents (models) a quantity whose rate of change is a percentage of the amount present (e.g., savings account).

Write the following questions on the board, give students a chance to answer them, and then call on pairs of students asking one student to give an answer and the other to verify the answer.

Questions: $\log_2(8) = ?$ $\log_4(16) = ?$ $\log_{10}(1{,}000) = ?$
$\log_{10}(1{,}000) = ?$ $\log_5(25) = ?$ $\log_3(25) = ?$

Work through the details of Exercise 21.

Small-Group Activity; Assign each group exercise 3.3/22. Call on a group to brief the problem to the class and then what-if it to exercise 23.

Suggested Homework: Study Section 3.4 (Shifting and Scaling Graphs)
Work Exercises 3.4/1, 2, 4, 5, 6

Lesson #22 Section 3.4 (Shifting and Scaling Graphs)
Two classes should be allocated for Section 3.4.

Objectives:

a. Understand how to shift a graph (horizontally and vertically)
b. How to scale a graph (including reflection)
c. Understand the role of asymptotes with respect to the graph of a function.

Suggested Lesson Plan
Return Quiz #4. Be positive in your comments.
Reflect over the past several lessons. Encourage the students to reflect and comment, starting with what is going well and concluding with what is not going well.

Ask for questions on the homework (direct computational questions to other students).

Small Group Activity: Assign an exercise to each group from the exercises 3.4/10-16. Ask one or two of the groups to brief the class on their work.

Class Activity: Call on individual students to work and then explain their reasoning to the class for each of the 19 parts in exercise 3.4/17-20

Suggested Homework: Review Sections 3.2 - 3.4
Work Exercises 3.4/7, 8, 23, 24

Lesson #23 Section 3.4 (Shifting and Scaling Graphs)
Second class on Section 3.4.

Objectives:

a. Understand how to shift a graph (horizontally and vertically)
b. How to scale a graph (including reflection)
c. Understand the role of asymptotes with respect to the graph of a function.

Suggested Lesson Plan
Ask for questions on the homework (direct computational questions to another student).

Small Group Activity: Exercises 25 and 26.

Use 3.4/25 to initiate a discussion on what can be inferred about a function from the existence (or non-existence) of an asymptote.

Ask students to give some real life scenarios showing asymptotic behavior (e.g., Example 3.4.2, volume of air in a balloon, amount of moisture in a sponge left on a counter).

Suggested Homework: Study Section 3.5 (Algebra of Functions)
Work Exercises 3.5/1, 4, 5, 12, 15

Lesson #24 Section 3.5 (Algebra of Functions)

Objectives:

a. Understand how to add, subtract, multiply, divide, and compose functions (graphically, numerically, and symbolically).
b. Understand Inverse Functions.

Suggested Lesson Plan

Quiz #5 (10-15 minutes) Suggested Problems: Do not use a calculator.

a. Evaluate $\log_2(16)$.
b. Sketch the graph of $f(x) = (x+2)^2 + 3$.
c. Shift the graph of $f(x) = 2^x$ such that y=2 is an asymptote of the shifted graph.
d. Problem similar to 3.3/4 (recognizing functions from their graphs).

Ask for questions on the homework (direct computational questions to another student).

Small Group Activity: Each group works Exercise 3.5/23. Ask one of the groups to brief their work to the class.

Small-Group Activity: Each group works Exercise 3.5/18. Ask one of the groups to brief their work to the class.

Suggested Homework: Review for Test #2

Lesson #25 Review for Test #2 (Sections 2.8-3.5)

Suggested Lesson Plan

Return Quiz #5. Be positive in your comments. Foster a positive, can-do attitude in the class.

Ask students what they did for review. Respond to questions from students' review

Ask individual students to illustrate/comment on each of the Objectives in Lessons #14-24

As time permits, students work problems from the following list:
2.8/5, 12 (set up only), 12 (solve); 2.9/5d, 17; 2.10/2b, 13; 3.1/14; 3.2/5, 28; 3.3/2, 4; 3.4/12, 18, 19; 3.5/12, 15

Suggested Homework: Prepare for the test

Lesson 26 **Test #2** (Sections 2.8-3.5)

Suggested Problems: (Select five including *b* and *i*.)

a. Solve the system $\begin{cases} 2x + 3y = 6 \\ x - 2y = 4 \end{cases}$.

b. Sketch the graph of the region defined by $\begin{cases} y > x \\ y < -x + 4 \\ x > 0 \end{cases}$ and then determine if the point (2, 3) is in this reason. Explain your reasoning.

c. Solve for the zeros of $f(x) = 2x^2 - x^3$.

d. Determine the range and domain of $f(x) = \sqrt{1 - x^2}$.

e. Give an example of a relation that is not a function. Explain why your relation is not a function.

f. Illustrate and describe the effect of increasing the coefficient *c* in the surge function $f(t) = ct2^{-2t}$.

g. A matching problem (matching function to graph) similar to 3.3/2 or 3.4/20.

h. Shift and/or reflect the graph of $f(x) = x^2$ such that the maximum point will be at (-2, 4).

i. Assume that an expanding oil slick, resulting from an oil spill in a bay, has a circular shape whose radius is a function of time, $r(t) = \sqrt{2t + 1}$. The radius is measured in feet and the time, *t*, in minutes. How long will it take before the area of the oil slick is 100 square feet?

Suggested Homework: Study Section 3.6 (Graphical Approximations)
Work Exercise 3.5/22
Work Exercises 3.6/3, 7, 8

Lesson #27 Section 3.6 (Graphical Approximations)
Two classes should be allocated for Section 3.6.

Objectives:

a. Understand how to solve an equation.
b. Understand how to graphical fit a curve to a scatter plot.
c. Understand and employ the iterative process (see Goal 4: Self-confidence in the Preface): "Try something, note the errors, modify previous attempt to lesson the errors and try again."

Suggested Lesson Plan
Ask a student to brief the class on his or her work on the homework problem 3.5/22 (surge function?). Ask if anyone had a different answer or used a different approach.

Ask a student to brief the class on his or her work on the homework problem 3.6/8 (Stopping distance). Ask if anyone had a different answer or used a different approach. Discuss how realistic the model is for slow speeds. Note that the stopping distance at zero speed is zero feet. If students incorporated this observation into their model, congratulate them and ask them to talk about their reasoning. If they did not incorporate this observation into their model, ask them why they did not.

Small-Group Activity: Ask each group to graphically fit a curve to the handshake data from Lesson #1 and then brief the class on what they did (e.g., show the list of their attempts.)

A portion of the data is:

Number of People	Number of Handshakes
1	0
2	1
3	3
4	6
5	10
6	15

Suggested Homework: Study Section 3.6 (Graphical Approximations)
Work Exercises 3.6/1, 5, 6, 11, 14. In Exercises 11, keep a record of your attempts.

Lesson #28 Section 3.6 (Graphical Approximations)
Second day on Section 3.6.

Objectives:

a. Understand how to solve an equation.
b. Understand how to graphical fit a curve to a scatter plot.
c. Understand and employ the iterative process (see Goal 4: Self-confidence, in the Preface): "Try something, note the errors, modify previous attempt to lesson the errors and try again."

Suggested Lesson Plan

Ask a student to brief the class on Homework Exercise 3.6/11. The briefing should include a listing of the attempts to fit a curve to the scatter plot as well as the reasoning involved. The primary purpose of the briefing is to call attention to the iterative process (see Goal 4: Self-confidence in the Preface): "Try something, note the errors, modify previous attempt to lesson the errors and try again."

Ask for questions on the homework (direct computational questions to another student).

Small-Group Activity: Assign each group to work 3.6/13. Emphasize the importance of the iterative process and ask each group to keep a record of their attempts to fit a curve to their scatter plot. Call on a group to brief their work to the class. Their brief should include their list of attempts and their reasoning for each attempt.

If time permits, repeat the previous Small-Group Activity with Exercise 3.6/16.

Suggested Homework: Study Section 3.7 (Symbolic Approximation of Data)
Work Exercises: 3.7/1(a,c), 2c, 5

The following data on the world's population was extracted from a U. S. Census Bureau report. Develop a model for the world's population based on this data and then determine whether or not 6.68 billion is a reasonable estimate of the world's population in 2008.

Year	World Population (billions)
1960	3.03
1970	3.71
1980	4.45
1990	5.27
2000	6.07

Assign a small-group Fun Project (e.g., FunProject.3.2) that is due on Lesson 32. (Feel free to assign different projects to different groups.) Let the students know that they conclude their Fun Project with a written report and a class briefing. Remind them of the point value that you have assigned to this activity. Talk to the class about your expectations and the opportunity to "show case" their work.

Lesson #29 Section 3.7 (Symbolic Approximation of Data)

Two classes should be allocated for Section 3.7.

Objectives:

a. Understand regression in term of minimizing the sum of squares of errors.
b. Understand how to compute a linear regression model of the form $y = mx$.
c. Understand how to use the regression programs in the calculator.

Suggested Lesson Plan

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises.

Ask a student to brief the class on Example 3.7.1. It is important that students understand this Example. Watching and listening to a student brief this Example, will help you gauge student understanding and where the difficulties lie.

Tell the students that the phrase "best fit," unless specified otherwise, refers to the regression curve that minimizes the sum of the squares of the errors. If one were to use the absolute value of the error rather than squaring the error, a different but equally acceptable result would be found. Part of the reason for using the sum of squares of errors method is that it is easier to minimize a sum of quadratic terms than it is to minimize a sum of absolute value terms.

Small Group Activity: Assign Exercise 3.7/7 to each group. After a suitable time has elapsed, ask one of the groups to brief the exercise to the class.

Small Group Activity: Assign Exercise 3.7/9 to be done using the method illustrated in Example 3.7.3 (Exact Polynomial Fit).

Conduct Peer Review #3 (5 minutes, students fill in a peer review matrix).

Suggested Homework: Consult your calculator manual to learn how to compute regression functions using your calculator.
Work Exercises 3.7/10, 17

Lesson #30 Section 3.7 (Symbolic Approximation of Data)
Second day on Section 3.7.

Objectives:

a. Understand how to use the regression programs in the calculator.
b. Understand how to obtain an exact polynomial fit to a scatter plot.

Suggested Lesson Plan
Return and discuss Test #2.

Look for examples of student reasoning that you can compliment and share with the class.

Reflect on the progress that students have made with respect to the course goals. In particular, note examples of improved problem-solving, student engagement, communication, and student attitude. Invite students to add their reflections to yours.

Small Group Activity: (Skip this activity if it was done in the previous class.) Exercise 3.7/7 to each of the groups and ask them to approximate the salinity percentage that would reduce the freezing point to -3° C. Compare the results from the different groups.

If time permits, Small Group Activity Exercise 3.7/8

Suggested Homework: Study Section 3.8
Work Exercises 3.8/1, 2, 4, 6

Lesson #31 Section 3.8 (Optimization)

Objectives:

a. Understand the role of the objective function.
b. Understand how constraint equations can be used to reduce an objective function to a function of one variable.
c. Understand how to graphically approximate the maximum and minimum values of a function of one variable.
d. Understand the problem-solving process that is outlined in the Section.

Suggested Lesson Plan

An interesting development to share with your students is that optimization used to be a topic reserved for calculus. However the availability of graphing calculators and computer graphing programs, has made optimization a college algebra topic. What is needed for optimization is the ability to model the situation and then determine the maximum or minimum value algebraically or from the graph of the model.

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises.

Assign each homework problem to a student with instructions to write their solution on the board and then brief it to the class. (If a student was not able to solve a problem, have the person put their attempt on the board and then let the class finish the problem.)

Ask about the Fun Project. Remind students of the due date and encourage students to work on their Fun Project.

Small Group Activity: Assign different problems to different groups. Select the problems from Exercises 3.8/7-14. Ask each group to brief the class on their problem. Check to see that each group follows the problem-solving process outline in the Section.

If time permits, assign Exercise 3.8/16 to the groups. This is an ill posed problem that requires students to define an objective function and make explicit assumptions. This exercise should lead to an interesting discussion. For example, in determining how the canvas should be position the question becomes do you want a high, short tent or a low, long tent. The high, short tent would provide for more comfort, however the low, long tent would provide for a lower profile that might be an important security consideration.

Suggested Homework: Review Sections 3.6-3.8

Announce Quiz #6 for the next class

Lesson #32 Review Chapter 3

Objectives:

a. Understand the function concept, the basic functions, and how to manipulate functions.
b. Understand the iterative process for graphically fitting a curve to a scatter plot.
c. Understand regression analysis with respect to minimizing the sum of the squares of errors.
d. Understand how to work optimization problems.

Suggested Lesson Plan

Quiz #6 (10-15 minutes) Suggested questions:

a. Include a question from Test #2 from Section 3.6.
b. Fit a curve to the data: (1, 5), (3, 9), (4, 11), (6, 15).
c. (Section 3.8/14) A 20-foot rain gutter is formed from a 10-inch strip of aluminum 20 feet long by pending up the sides of the strip to form a U shape 20 feet long. (The sides are bent perpendicular to the bottom.) Determine the width of the sides and bottom that will maximize the volume of the rain gutter.

Ask about the Fun Project. Remind students of the due date and encourage students work on their Fun Project.

Assign problems that address the Lesson Objectives to the groups to work and brief to the class. Assign the same problem to two or three different groups to enable comparisons. There should be time enough for each group to work two problems.
Sample problems:

a. Section 3.3/16, 17
b. Section 3.4/9
c. Section 3.5/15
d. Section 3.6/15
e. Section 3.8/17
f. Section 3.8/18

Suggested Homework: Finish the Fun Project and prepare to brief it to the class.

Lesson #33 Fun Project Day

Objectives:

Students brief their Fun Project

Suggested Lesson Plan
Provide time for each group to brief their Fun Project.
Make this into a "big thing." Invite other faculty to sit in.

Suggested Homework: Study Section 4.1 (Mathematical Modeling)
Work Exercises 4.1/1, 4, 9; Answer Queries 1 and 2.

Lesson #34 Section 4.1 (Mathematical Modeling)

Objectives:

a. Understand the Problem-Solving/Modeling Process as applied to Example 4.1.1.
b. Understand how to sum a geometric series.
c. Understand how to model Option 2 of Example 4.1.1 using a recursive sequence.

Suggested Lesson Plan
Return Quiz #6. Be positive in your comments, both with respect to the quiz and the project presentations. Ask the faculty who sat in on the project briefings for comments that you can pass on to the class.

Reflect over the past few lessons including the Fun Project (content, student initiative, student attitude, class work). Encourage students to reflect and share their reflections.

Conduct Peer Review #4 (5 minutes, students fill in peer review matrix).

Ask for questions on the homework (direct computational questions to other students).

Work through the development of the summation formula for a geometric series. Present the students with examples to work.

Be sure that students understand Example 4.1.4.

Small Group Activity: Jeff celebrates his twenty-first birthday by drinking three cans (12 ounce) of beer in one hour. Jeff weighs 120 pounds, so after his third can his Blood Alcohol Concentration (BAC) is 0.084 (computed using the Wisconsin DOT BAC calculator). Assume his body metabolizes alcohol at a rate of 0.01 per hour from the time he began to drink the first can. How long will it take before Jeff is legally able to drive? (A person with a BAC of 0.08 or higher is legally drunk.)
Each group works this problem and then one briefs the class explaining their reasoning. Ask another group to what-if the problem (e.g., BAC for 120 pound female who consumes three drinks in one hour is 0.102). This problem is similar to Example 4.1.4.
Source http://www.dot.wisconsin.gov/safety/motorist/drunkdriving/calculator.htm.

Be sure that students understand how the symbolic solution, $a(n) = (1+r)^n a(0)$, for the recursive sequence $a(n) = a(n-1) + ra(n-1)$ is obtained.

Suggested Homework: Study Section 4.2 (Modeling (Business))
Work Exercises 4.2/1, 2, 5, 7

Lesson #35 Section 4.2 (Modeling (Business))
Two classes should be allocated for Section 4.2.

Objectives:

a. Understand how to formulate a recursive sequence model for savings accounts, credit card debts, car loans, mortgages, etc.
b. Understand how to iterate a recursive sequence.
c. Understand the properties of exponential and logarithmic functions

Suggested Lesson Plan
Assign a problem similar to the parts in 4.2/1, 2 to each student to work and then brief to the class.

Ask students what they learned from their homework? Probe students to talk about what they gained personally and intellectually from the assignment rather than just referring to exercises. Ask for questions on the homework (direct computational questions to other students).

Small Group Activity: Assign Exercise 4.1/18 to each group to work and brief to the class.

Work through Example 4.2.1. Make a judgment call on the expected level of understanding of the development of the symbolic solution of Sue's Plan. Instruct students to note (underline, make marks in the margin, etc.) the general solution to the recursive sequence model, $a(n) = (1 + r)a(n - 1) + d$. This is the basic recursive sequence model that will be used in most of the applications. As a way of building student confidence in the general solution, assign values to r, $a(0)$, and d., then have the students compute the value for $a(5)$ and verify it by iterating the recursive sequence.

Show the class how to iterate using a graphing calculator.

Work through Example 4.2.2, showing how the recursive sequence model is adapted from the model in Example 4.2.1.

Work through Example 4.2.3, showing how the recursive sequence model is adapted from the model in Example 4.2.1.

If time permits, ask the groups to work Exercise 4.2/5. Ask someone to explain the recursive sequence model given in the exercise.

Suggested Homework: Study Section 4.2 (Modeling (Business))
Work Exercises 4.2/8, 9, 13, 14

Lesson #36 Section 4.2 (Modeling (Business))
Second class on this Section.

Objectives:

a. Understand how to formulate a recursive sequence model for savings accounts, credit card debts, car loans, mortgages, etc.
b. Understand how to iterate a recursive sequence.
c. Understand "doubling time."

Suggested Lesson Plan
Assign a homework problem to each group to brief to the class. Insist that the problem is set up correctly and that the students explain how they developed their model.

Small-Group Activity: assign a problem to each group to work and then brief to the class. Select problems from the Exercises 4.2/18-24. Insist that the problem is set up correctly and that the students explain how they developed their model.

Suggested Homework: Review for Test #3

Lesson #37 Review (Sections 3.6-4.2)

Suggested Lesson Plan
Ask students what they did for review.

Respond to questions from students' review

Ask individual students to illustrate/comment on each of the Objectives in Lessons #27-36

As time permits, ask students to work problems from the following list:
3.6/5, 11, 14; 3.7/2c, 16; 3.8/1, 12, 18; 4.1/2c, 4b, 11, 16; 4.2/2a, 5, 9, 24

Suggested Homework: Prepare for the Test #3

Lesson #38 Test #3

Suggested Problems (Select four problems.)

a. Is $x = 3$ a zero of $f(x) = x^3 - 3x^2 + 2x - 6$? Explain you reasoning.

b. Given the recursive sequence, $s(n) = s(n-1) + n$, $s(0) = 0, n = 0,1,2,3,...$, create a two-column table (left-hand column labeled *n*, right-hand column labeled $s(n)$) for *n* from 1 to 5. Is the data given by your table linear? Explain.

c. Illustrate the linear regression process by fitting a line that passes through the origin to the scatter plot consisting of the three points: (1, 3), (3, 2), (4, 6). Draw a conjectured line on the scatter plot of the data and then indicate the point errors. Form the error function and determine the slope of the line that minimizes the error.

d. Jacob wants to determine how high his rocket will fly. However, his height sensor has limited range and only yields the following data for his test flight.

Time in Seconds	Height in Feet
1.0	50
1.3	61
2.3	85
4.1	78
5.1	46
6.0	0

Develop a model based on this data and then determine how high the rocket went.

e. Cereal companies offer their product in different size boxes. However due to the limitations of horizontal self space in food stores, they generally keep the thickness of their boxes constant and only changes the height and length of the boxes. Develop a model for doubling the volume of a cereal box that has a height of 10 inches and a length of 7 inches.

f. Malcolm has accumulated a debt of $2,000 on his credit card. The annual percentage rate (APR) charged by the credit card company is 18%. Malcolm decides to tear up his card in order not to be tempted to make additional charges. He also decides to allow the monthly finance charges to accumulate until he graduates in three and one-half years. How much will Malcolm owe the credit card company when he graduates?

Lesson #39 Reflection on the Semester

Objectives:

a. Return and comment on the Fun Projects.
b. Return and comment on Test #3

Suggested Lesson Plan

Return and comment on the Fun Projects.
Return and comment on Test #3.
Look for reasons to be upbeat and complimentary. Speak about the class's success in addressing the course goals. Speak about individual efforts. Speak about attitudes. Speak about group work.

Conduct a class/individual reflection starting with the last block of Lessons (#27-36) and then expanding to cover the full semester. A sampling of questions that could be used to "pump prime" the discussion include:

- What was your best experience in this class?
- What was your worse experience in this class?
- Did you make progress toward becoming an exploratory student as a result of this class? Describe and illustrate with specific experiences.
- Did you make progress toward taking more responsibility for your own learning as a result of this course? Describe and illustrate with specific experiences.
- Have your communication skills improved as a result of taking this course?
- Has there been a change in your self-confidence for doing mathematics as a result of taking this course?
- Has your interest in mathematics changed as a result of taking this course?
- Has your attitude toward mathematics changed as a result of taking this course?
- Did you learn how to read and study the text?
- How successful were you as a group member? (What type of grade would you give yourself for your group work?)
- Did the assignments and class work address the course goals?
- Were the assignments and class instructions clear to you?
- Where did you go to get help when needed?
- Did you join a study group?
- How confident are you that you will pass this course?
- What additional math course(s) do you plan to take?
- Would you recommend to a friend that he/she take this course rather than the traditional college algebra course?
- What suggestions do you have for improving this course?

Do not feel limited to these questions and do not allow a student to just answer Yes or No, but encourage them to elaborate. Keep the atmosphere positive.

Suggested Homework: Review for Final Exam.
Assign each group a portion of the course for which they will be responsible for preparing and conducting a class review.

Lesson #40 Review for the Final Exam

Objectives:
a. Student Review for the final exam.
b. Develop a positive and confident attitude going into the final exam.

Suggested Lesson Plan
The review should be student led with you serving as a "last resort" in answering questions. How successful the students carry out the review is a measure of how much they have matured in the class.

Conduct Peer Review #5 (5 minutes, students fill in peer review matrix)
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Suggestions of Final Exam Questions
1. (Lsn 1, Obj. b) 2.4/5 (scatter plot only, include graph paper).
2. (Lsn 3, Obj. b) Fill in the blank in the following statement. At a coat sale that advertized ____ percent off, Eve bought an $80 coat for $60.
3. (Lsn 4, Obj. a) Sam bought desserts for ten people, including himself. One dessert cost $3, five desserts cost $2 apiece, and four desserts cost $1 apiece. Sam told his friends that the desserts cost $1, $2, and $3. So the average cost is $2. If each of you gives me $2, we'll be even. Did Sam gain, lose, or break even on this arrangement? (See 2.2/14)
4. (Lsn 4, Obj. b) 2.2/5
5. (Lsn 8, Obj. d) 2.7/4
6. (Lsn 14, Obj. b) 2.8/12
7. (Lsn 15, Obj. e) 2.9/17
8. (Lsn 17, Obj. e) Graphically approximate the zeroes of $f(x) = 2^x - x^2$
9. (Lsn 19, Obj. b) Matching functions and graphs (e.g., 3.3/2, 4 or 3.4/19)
10. (Lsn 20, Obj. b) Identify functions from their graphs (e.g., 3.4/20)
11. (Lsn 28, Obj. c) 3.6/10 In what year would the production first exceed 300,000 vehicles?
12. (Lsn 30, Obj. c,d) 3.8/4
13. (Lsn 30, Obj. c,d) 3.8/17
14. (Lsn 33, Obj. a) 4.2/24
15. Describe the central question in your Fun Project.

Sections 4.3-4.8 present discipline specific situations that can be addressed with recursive sequence models. These sections are included to show the breadth of college algebra modeling as well as providing for honor or special project work. When assigning one of these sections, you are encouraged to collaborate with a colleague in that discipline.