A MODELING-BASED COLLEGE ALGEBRA COURSE AND ITS EFFECT ON STUDENT ACHIEVEMENT

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ABSTRACT: In Fall 2004, Virginia Commonwealth University (VCU) piloted a modeling-based approach to college algebra. This paper describes the course and an assessment that was conducted to determine the effect of this approach on student achievement in comparison to a traditional approach to college algebra. The results show that compared with their counterparts in the traditional sections, students in the model-based approach had a higher success rate in the college algebra course; performed better on common final exam questions; performed less well in a subsequent precalculus course and slightly better in a subsequent business mathematics applications course. Overall, by the end of the following semester, the percentage of students who enrolled in the modeling course and successfully completed college algebra and a subsequent course required for their majors was significantly larger than the corresponding percentage of students in the traditional sections.

KEYWORDS: College algebra reform, mathematical modeling, assessment, student achievement, student retention

INTRODUCTION

For more than four decades, the focus of a traditional college algebra course has been to prepare students for calculus. According to the Fall 2000 survey of the Conference Board of the Mathematical Sciences [2], 211,000 students enrolled in a course entitled "College Algebra" in that semester. This is a 32% increase over the last 20 years in the enrollment in a college algebra course offered by colleges and universities across the country. The figure would be much larger if it included students enrolled in courses that include traditional college algebra topics in their curriculum but may cover other topics as well and, therefore, are labeled with a different title other than "College Algebra". College algebra is a significant milestone in the educational experience of many college students most of whom are not planning a career which requires knowledge of calculus. As a result, the curriculum of the typical college algebra course may not be appropriate for most students.

College algebra reform is taking place in schools across the country. Mathematical organizations are engaging mathematicians and educators in discussions on the role of college algebra in the curriculum and the educational experiences that are most meaningful for students in the course. In particular, the Committee on the Undergraduate Program in Mathematics (CUPM) recognizes the "mismatch between the original rationale for a college algebra requirement and the actual needs of the students who take the course" [1, p. 27] and provides recommendations for a meaningful experience for future college algebra students. CUPM advocates that mathematics departments make changes to ensure that entry-level mathematics courses provide worthwhile quantitative experiences for students who do not aspire to a career requiring higher level mathematics while preparing those who will take courses beyond the introductory level. A significant amount of time in an entry-level mathematics course, like college algebra, should be spent on in-depth study of a smaller number of topics [1]. This type of experience is more meaningful and more applicable to other courses students take when compared to giving students cursory exposure to a wide variety of mathematical concepts. The course should focus on mathematical concepts that are important to other academic disciplines such as business and biology and help students develop the ability to communicate quantitative ideas orally and in writing [1].

With these ideas in mind, Virginia Commonwealth University (VCU) piloted a modeling-based approach to College Algebra in Fall 2004. The goals of the new approach were to offer a course that: (1) Focused on mathematics topics that are important in other disciplines, developed abilities of students to work as a member of a team and to communicate quantitative ideas orally and in writing, and emphasized the development of mathematical models and the use of technology; (2) Increased the percentage of students who successfully completed college algebra; and (3) Did not decrease the ability of students who passed the course to complete core algebraic computations and applications of the type included in a traditional college algebra textbook.

The chair of the Mathematics Department along with the Department Curricular Committee requested that the author of this paper design a research program to determine the impact of the pilot project. As described in the paper: (1) A modeling-based course with the modeling, collaborative, and communication features described was offered to 284 students in eight sections during the same semester that 989 students enrolled in 28 sections of a traditional college algebra course. (2) A grade of A, B, or C was earned by 71.83% of the students enrolled in the modeling sections and by 49.70% of those in the traditional sections. Embedded in these statistics is the fact that 89.6% of the students in the modeling sections took the final exam (5.63% withdrew and 4.77% of others did not take the final exam). On the other hand only 71.33% of the students in the traditional sections took the final exam (20.34% withdrew and 8.33% of others did not take the final exam). (3) There is no evidence that the greater emphasis on modeling, collaboration, and communication resulted in a diminution of the ability of students to complete core algebraic computations and applications.

Students in the modeling sections and those in traditional sections were administered a common portion of the final exam that covered algebraic computations and applications. The performance of 89.6% of students in the modeling sections who took the final exam was comparable (indeed somewhat better) to the 71.33% of the students who took the exam in the traditional sections. Details of the two approaches, the design of the study, and possible explanations for the superior performance of the students enrolled in the modeling sections are outlined below.

BACKGROUND

VCU is an urban institution with a population of 28,500 undergraduate and graduate students over two campuses (academic and medical) in close proximity. Until Fall 2002, college algebra was a self-paced course in which students attended class with an instructor and several tutors, who provided help as the students worked independently on mastering the course material and preparing for five tests which made up the course grade. In 2002, due to extensive cuts in the University's budget, college algebra was taught in a lecture format with class sizes of 180 to 240 students. In addition to three semester hours of lecture, students also met in a computer lab one hour a week to work on assignments completed with a software package.

An instructor and tutors were available during this hour to answer questions. Students also had access to the software program and could complete assignments at home. While these homework assignments were required, the majority of the class grade was based on four tests and a comprehensive final exam. Attendance in both the lecture and lab was poor, except for test days. The ABC rate for college algebra in this format was 36%. This rate is similar to the rate achieved by many colleges and universities across the country in which skill-based algebra courses are offered. The low passing rate coupled with the fact that 25% of students fail to complete the course was strong support for change in our college algebra program.

In Spring 2004, VCU secured a Title III U.S. Department of Education Strengthening Institutions Development Grant to improve student outcomes in the mathematics and science courses that serve as a "gateway" to the traditional college experience – introductory courses in mathematics, chemistry, and biology. The mathematics department decided to pilot a major change to college algebra. A committee of interested faculty members held meetings to discuss the focus of the to-be-designed course, gathered feedback on college algebra from faculty in departments with courses for which college algebra is a prerequisite, and surveyed a wide variety of textbooks. The committee, with the support of the department, felt strongly that we should use this opportunity to pilot a course that was a major shift from the college algebra course of the past.

We decided to design a modeling-based course that would expose students to mathematics in light of real-world applications and would introduce algebraic skills when they were needed to solve a particular problem. Two sections of college algebra served as pilots in Spring 2004 using *Earth Algebra* [3] and focusing on helping students develop mathematical models as opposed to highlighting algebraic skills. The department was pleased with the results and decided to expand the pilot to include a larger number of sections in Fall 2004. We invited the author of *Contemporary College Algebra: Data, Functions, and Modeling* (CCA) [4], Professor Don Small of the United States Military Academy, to visit VCU to discuss a modeling-based approach to college algebra.

In Fall 2004, eight sections of college algebra consisting of 35 students each were taught using CCA [4]. The remaining 27 sections of college algebra were also limited to 35 students with instructors lecturing three hours a week on traditional college algebra topics. The same computer lab component that had been used in previous semesters was also included in the traditional sections. The modeling sections met one hour a week in the computer lab as well, but the assignments were significantly different from the assignments completed in the traditional sections. While major changes were made to how the modeling sections were taught, a significant change – reduction in class size – was also made to the traditional sections of college algebra. Comparison and analysis of these two approaches to college algebra is outlined below.

COURSE DESIGN

Class Participants

Yearly, 2,000 students enroll in college algebra at VCU. Students taking the course in Fall 2004 covered all academic years – freshmen through seniors. The distribution of academic years was the same for both versions of college algebra with the largest proportion, 80%, being freshmen. College algebra is a prerequisite for precalculus and a mathematics applications course for business majors at VCU. Therefore, for many students enrolling in college algebra, this was not their last required mathematics course. Most of the students enrolled in Fall 2004 majored in a science or business field but the class consisted of participants from a wide variety of majors. Students took the mathematics placement test to determine the college mathematics course for which they were prepared. Based on the placement level, all students entered college algebra with a similar level of understanding of mathematical skills and concepts at the beginning of the course. They registered for the class based on their individual schedules and seat availability. All sections of college algebra were limited to 35 students. The registration process at VCU did not allow for random assignment into the modeling and traditional sections but students were not aware of which sections would be taught using the modeling approach beforehand.

Modeling Approach

The instructors of the eight modeling sections were each assigned two teaching assistants to help with class-related activities. The assistants attended all class meetings where they helped students who were having difficulties and helped facilitate group activities. Their out of class duties included, but were not limited to, grading course assignments, tutoring students who needed extra help, and running help sessions before each test. The modeling section instructors met weekly to discuss the week's activities and make curriculum decisions including what topics to cover next, the assignment of homework and projects, and the questions to include on assessment instruments (i.e. quizzes and tests). Each instructor was given license to run the course and cover the topics as he or she chose. However, through the weekly meetings it was clear that, except for minor deviations, each section was run in a similar manner. An attendance policy was implemented in many (but not all) sections stating that students with more than 6 unexcused absences would receive a grade of C or below in the course and that students with more than 9 unexcused absences would be dropped from the course. Tardiness was addressed with the policy that a student's final grade would be reduced by 1% for every 5 times the student arrived late or left class early.

While the percentages may have been slightly different for some, Table 1 describes a general syllabus for the modeling-based college algebra class. The final grade was composed of:

Homework	5%		Quizzes	15%
Class Activities	10%		Tests	30%
Group Projects	20%		Final Exam	20%

Table 1. Distribution of Final Grade by Class Activities

Homework assignments were given daily, but were not graded on a regular basis. Most often students provided answers to homework problems on the chalkboard at the beginning of class allowing for review of the previous material and an opening to new material to be covered in the day's activities. Class activities were assigned every day and a high percentage of them were graded by the course instructor or teaching assistants.

The typical format for a 75 minute class period was a 5 − 10 minute review of old material through homework problems followed by a 10 − 15 minute introduction to new material with motivation and examples provided by the instructor. The remaining 50 minutes consisted of students working on modeling problems in groups of 2 − 4 with intermittent pauses for whole- or partial-class discussion on issues that would arise or skills that needed to be reinforced while students were working. An example of a modeling problem and the process used to arrive at the mathematical model is outlined below. Mathematical skills were covered on the day they were needed for a given application. This provided context for students on the importance of being able to apply a particular mathematical skill. The use of graphing calculators to evaluate problem situations was emphasized on a daily basis. On most days, at the end of the class period, students turned in their group in-class activity sheet for a grade. The computer lab component was 50 minutes in duration. Quizzes were given during these time periods. This time was also used to reinforce concepts covered in the 75 minute class periods by allowing students to delve into more detailed modeling problems provided on handouts or through the software program that accompanies

CCA [4]. Early in the semester, computer lab time was spent with students practicing the organization of data through the table and graph capabilities of a computer spreadsheet.

Students completed two group projects throughout the semester. These were designed for students to explore the role of mathematics in the real world. Students worked with 3 – 4 in a group and were given 2 – 3 weeks to complete the project. In most classes, students had a choice of 2 – 3 projects. The projects required students to gather data then find a mathematical model (linear, quadratic, exponential, etc.) that most appropriately fit their data. Their typewritten reports included (1) a narrative of the project details and the appropriate tables and graphs to support their results, (2) a list of group meetings and the role each member played in the group, and (3) a paragraph summary of what they learned by completing the project. The students used graphing calculators and computer spreadsheets to organize and analyze their data. A brief description of two projects is provided below. All projects in the course were assigned from CCA [4].

1. Research Mildred Trotter's work in predicting the height of a person based on the lengths of certain bones. Gather data on the head length, forearm length, stride length, and height of 20 students. Use linear functions to determine the line of best fit for height versus each of the other three categories. Determine which model will provide the best prediction of a person's height.

2. Research the history of postage stamps in the U.S. and collect and evaluate data on the number of stamps issued each year. Plot the data in a scatter plot and determine an appropriate function to model the data. Compare the actual number of stamps issued in a particular year with the value determined by the mathematical model.

Six quizzes were given throughout the semester. They each consisted of $4 - 6$ questions and were designed to be completed in $20 - 30$ minutes. The questions were mostly modeling-based with at most one skill question on each quiz. Students took quizzes during the computer lab hour of the course. The tests were written by the lead instructor for college algebra with suggested changes from other instructors given during the weekly meetings. Seventy percent of each test was devoted to modeling-based questions and the remaining 30% assessed mathematical skills. Individual instructors (with the help of teaching assistants) graded the exams for their class. The final exam was comprehensive and had the same distribution of modeling and skill-based questions as the other tests. The entire skill section of the final exam and three of the eleven modeling questions were identical to questions that appeared on final exam taken by students in the traditional sections of college algebra. The questions common to both exams were used to compare the achievement of students in the modeling and traditional sections.

The modeling approach to college algebra covered topics similar to those in a traditional college algebra course but the emphasis was on different aspects of the topics. Skills were covered only when needed to apply to a particular mathematical model. While most of the traditional course was devoted to skill development, less that 25% of the modeling course had the same focus and, therefore, many traditional skills were not included in the curriculum. For example, the modeling instructors did not spend time on rationalizing function denominators or covering the rules for factoring special polynomials. A considerable amount of time was spent using graphing calculators in finding and evaluating mathematical models. Graphing calculators were a central part of instruction from the first day of class and students were very familiar with the calculator's graphing features by the end of the course.

The semester began with several class sessions on measures of central tendency and appropriate ways to display data (histogram, pie chart, etc.). This was followed by analysis of the role of mathematical variables in the development of mathematical models with specific examples from algebraic and geometric sequences. To facilitate study of the relationship between variables, properties of linear equations and systems of equations were introduced with particular emphasis on the significance of rate of change (i.e. slope). Storing and evaluating data with matrices was used to analyze a variety of applications that required linear equations.

The second half of the course focused on developing an understanding of function and helping students determine the appropriate mathematical model for a variety of real-world situations. Graphs, tables, and symbolic expressions were used to study the definition of function as well as the properties of functions (increasing, positive, maximum value, etc.). A significant amount of time was spent helping students construct a mental library of the major families of functions (linear, quadratic, radical, exponential, etc.). To successfully apply a function family to a problem situation, time was spent understanding the change that numerical values have on symbolic expressions and graphs. Transformations (shifts and stretches) are of particular importance to a modeling approach to college algebra. Students spent a considerable amount of time analyzing real-world data, applying a mathematical model to the data, and making predictions based on their model and the data. The mathematical model was arrived at through a series of approximating functions which students determined through group discussion and exploration.

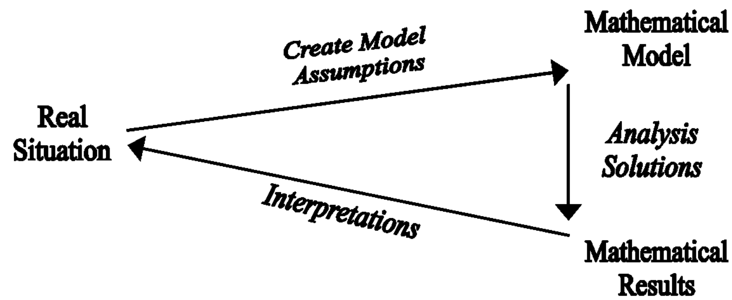

Figure 1. The Modeling Process.

A Mathematical Modeling Example

The modeling process is best illustrated through the diagram in Figure 1. This diagram is presented early in CCA [4] and is referred to often throughout the textbook. The problems are designed with the modeling process as a foundation for the work needed to arrive at solutions. For example, in one type of problem that is covered in the second half of the semester, students plot a set of points based on a real situation and use those points to develop a mathematical model. They check the model against the plotted data and refine the model through a series of approximation functions until it most accurately represents the data. Then they determine a "new" data point from the model and interpret its meaning in light of the situation upon which the model is based. Consider the example below.

Problem Situation:

The U.S. Census Bureau recently reported that in 2004 the foreign-born population in the U.S. reached 32.4 million. This number is 12% of the total population of the United States and is up from 9.3% of the total population in 1996. Table 2 lists the percentage of the U.S. population that was foreign-born in select years over the last seventy years.

Year	1930	1940	1950	1960	1970	1980	1990	1996	2004
Percentage	11.6	8.7	7.0	5.3	4.8	6.2	8.0	9.3	12.0

Table 2. Percentage of the U.S. Population that as Foreign-born in a Given Year.

Use this data to develop a mathematical model for the percentage of the U.S. population that was foreign-born in the last seventy years. Use the model to predict when percentage of the U.S. population that is foreign-born will reach 13%.

Modeling Process and Solution:

Students begin by creating a scatter plot of the data. By evaluating the shape of the data, they determine the function family (i.e. linear, quadratic, exponential, etc.) that will be most useful in developing a model that best fits the data. Early in the process questions arise on the appropriate graphing calculator window to use to display the data and the best way to represent years (1930 – 2004) in the window.

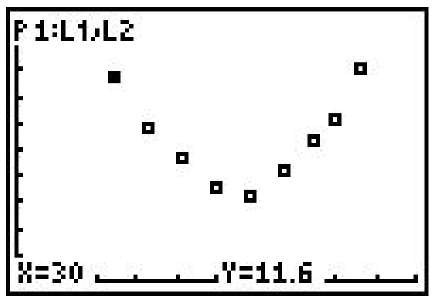

Figure 2. Scatter Plot of Foreign-born U.S. Population Data.

Students are encouraged to compare the scatter plot to their library of function families. Noting that the data has a distinctive quadratic shape, an appropriate model is based on the quadratic function family. Students consider the general equation $y = a(x - b)^2 + c$ and work in groups to determine numerical values for a, b, and c to create a model that best fits the data. Many student teams begin by locating a possible vertex. In this case, one possibility (but not the only possibility) is using the point $(70, 4.8)$ which represents the year 1970 when the percentage of the U.S. population that was foreign-born was 4.8% and is a "low" data point in the scatter plot. With this point as the vertex, the first approximating function is $y = (x - 70)^2 + 4.8$. (See Figure 3.)

Working with possible values for a yields the next approximating function which in Figure 4 is $y = .005(x - 70)^2 + 4.8$. Clearly no two teams will arrive at the same set of approximating functions and even based on Figure 3 there are many possibilities for the second approximating function.

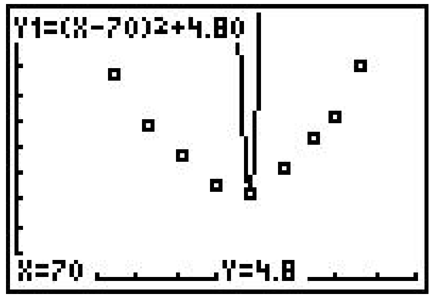

Figure 3. Scatter Plot and First Approximating Function.

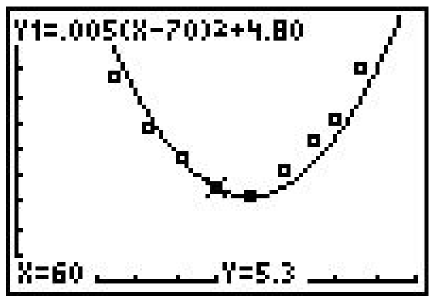

Figure 4. Scatter Plot and Second Approximating Function.

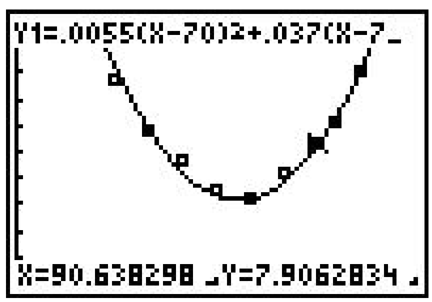

Figure 5. Scatter Plot and Final Approximating Function.

Throughout this process, the instructor and teaching assistants walk around the room providing clarification, help, or hints to teams who need them. After adding in a middle term of the form $d(x-70)$ (where d is a real number) to get the correct tilt for the parabola and making slight adjustments to the numerical values a, b, c, and d each student team arrives at a final approximating function. In Figure 5 the function $y = .0055(x-70)^2 + .037(x-70) + 4.8$ is the mathematical model for the population data.

By using a graphing calculator, the function is in a useful form and there is no need to simplify it. Student teams are encouraged to compare their modeling function with other teams and decide which team most accurately represented the data with their model. Using the model, they predict the year in which the percentage of the U.S. population that is foreign-born will be 13%. Through the model developed above, 13% will be reached half way through 2005 (see Figure 6).

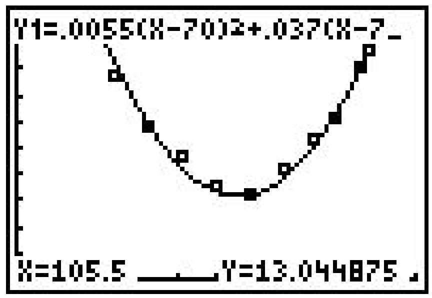

Figure 6. Analyzing a "New" Data Point Generated from the Mathematical Model.

Students are encouraged to question the accuracy of their model and discuss factors (mathematical and situation-based) that could generate the need to make changes to the model. Note that the method of finding a series of approximating functions requires constant checking of the functions against the original data. The modeling process outlined in Figure 1 is the foundation of this type of problem.

Traditional Approach

All of the traditional college algebra sections followed the same format: 150 minutes of lecture by an instructor and one 50 minute computer lab session per week. Attendance was not mandatory. *Intermediate Algebra* [5] was the textbook used for the course. Homework using the *Hawkes Learning Systems: Intermediate Algebra* [5] that accompanied the textbook was required work. Students were assigned a due date for the homework set that corresponded to textbook sections covered in the lectures. By the due date, they were to complete successfully at least 80% of the problems. All of the work was done using the computer software and the instructor only had access to the following: (1) a list of sections in which a student achieved mastery, (2) the date upon which mastery was achieved, (3) the time it took the student to reach the mastery level, and (4) the number of attempts the student needed to require mastery. The software is user-friendly and contains a set of practice problems and section tutorials for students who need extra help. The computer lab session was used for students to get help from the instructor or tutors and to work on achieving mastery on the homework problems.

All tests, except the final exam, were also given in the computer lab. Each test was of the traditional paper-and-pencil format and contained 15 – 20 free response questions. The tests were written by the computer math lab staff and all traditional sections of college algebra took the same tests. The final grade was based on five tests – four 50 minute exams and one final exam worth twice the value of each of the other tests. The homework assignments were required to pass the course but only figured into the final grade if the student had a passing average on the five tests. In this case, a student's final grade was based on the average of four tests, the homework (given the weight of one test) and the final exam (given the weight of two tests).

The topics covered in the traditional college algebra sections are those that appear in a typical college algebra textbook including: linear equations and inequalities, definition of function, operations on polynomials and rational expressions, in-depth study of quadratic equations, properties and equations involving exponential and logarithmic functions, and arithmetic and geometric sequences. *Intermediate Algebra* [5] features a skill-based approach to the topics with 10% of the textbook sections covered in VCU's traditional college algebra course devoted to application problems.

COURSE EVALUATION

To measure the success of the new approach to college algebra on student achievement, we gathered three forms of data: (1) grades given by instructors in all sections of college algebra in Fall 2004, (2) student answers to a set of skills and modeling questions that were identical on the modeling and traditional final exam, and (3) semester grades in Spring 2005 in courses for which college algebra is a prerequisite.

Course Grades

Table 3 contains the percentage of students receiving a grade of A, B, or C after completing a course under each form of instruction. These percentages are based on the grades of 284 students who enrolled in the modeling-based college algebra course and 989 students who enrolled in the traditional course.

Modeling Approach			Traditional Approach	
ABC	71.83		ABC	49.70
DFW	28.17		DFW	51.30

Table 3. Percentage of Students Passing Each Course for Fall 2004.

Based on a two sample test of proportions, the percentage of students getting an A, B, or C in the modeling-based course was significantly larger ($p < .01$) than the percentage of students getting one of these grades in the traditional courses. It should be noted that these courses were taught under very different syllabi and, therefore, the higher passing rate is not necessarily a reflection of student achievement. An interesting statistic is embedded in the DFW rates of these courses. The percentage of students who withdrew from the traditional college algebra course was 20.3% while the corresponding percentage for the modeling course was 5.6%. The number of students completing the entire semester is noteworthy. With a withdrawal rate that is typically 24% per semester in college algebra, we are pleased that 95% of the students enrolled in the modeling course were willing and able to complete the course.

While the ABC rate was significantly smaller than the passing rate for the modeling-based college algebra course, based on a two sample test of proportions the passing rate for the traditional course was significantly larger ($p < .01$) than the 36% of students who completed the large section skill-based course in previous three semesters at VCU. Therefore, the passing rate for the traditional approach reveals that the change made to those sections had a meaningful effect. It appears that more students will successfully complete a college algebra course taught in small sections with regular access to an instructor. More than just small class size is necessary for a successful college algebra course but, nevertheless, we are pleased with the significant increase in the number of students that successfully completed the traditional college algebra course.

Common Final Exam Questions

An important component of our evaluation of these two methods of teaching college algebra was the analysis of questions that appeared on final exams taken by students in both the modeling and traditional sections. The exam questions were written by the author, who was also a modeling section instructor, with input from instructors of modeling and traditional sections. The questions were approved by instructors of both types of sections before they were included in the exam. The ten skill questions consisted of the entire skill section of the modeling-based final exam and 41% of the skill section of the traditional exam. The questions were based on skills covered in both the modeling and traditional sections. They included skills such as: (1) find the slope of the line passing through a pair of points, (2) find the inverse function for a given function, (3) factor a third degree polynomial, and (4) determine the function resulting from the composition of two given functions.

The modeling problems consisted of three application problems written in a manner similar to problems found in a traditional college algebra textbook and were based on applications covered in both the traditional and modeling sections. While they spent much more time studying applications, students in the modeling sections did not have an unfair advantage based on the type of problems or the wording of the problems. The questions, each having two parts, were: (1) an application requiring the construction and evaluation of a system of linear equations, (2) a traveling baseball problem to determine the maximum height of the ball's path and time it takes the ball to hit the ground, and (3) a problem to determine the amount of money in an account with continuously compounding interest after a given amount of time and the amount of time to accumulate a given amount of money.

The analysis was conducted with all eight sections taught under the modeling approach and eleven randomly selected sections of the traditional course. Before the exams were graded by individual instructors, the pages containing the questions for the assessment were photocopied so clean copies were available for research purposes. Names and other distinguishing student information were not photocopied so the grader had no information about the individual student connected to each paper. The copies were organized according to class section and the sections were given non-distinguishing identification numbers so that the grader did not know whether he was grading a section of modeling or traditional papers. One person graded all nineteen sections of test papers. He used the same method of partial credit for all of the papers.

	Modeling Approach				Traditional Approach		
	N	Mean	St. Dev.		N	Mean	St. Dev.
All Questions	217	69.68	16.19		285	63.38	14.27
Skills Questions	251	67.05	10.58		285	60.59	9.10
Modeling Questions	217	74.29	7.64		285	68.49	7.47

Table 4. Analysis of Test Question Data by Approach to College Algebra.

Table 4 contains the means and standard deviations for all of the questions as well as separate values for the skills questions and the modeling questions. Seven modeling sections (217 students) completed the entire set of questions. The instructor of one modeling section changed one of the modeling questions on the final exam he gave to his class. As a result, the data for his class was not included in the analysis of the modeling questions or the entire set of test questions. The skill section of his final exam was unchanged, so the data for his students was included in the analysis of skills questions.

A two sample t-test for the data on the entire set of test questions shows that the average score for students in the modeling-based course was significantly higher ($p < .001$) than the average score for students in the traditional course. At the 1% level of significance, the results of a separate analysis of the skills questions and modeling questions statistically favored students in the modeling-based course as well. A nested two-factor ANOVA ($F = 2.84$, $p < .01$) was used to analyze the data from all questions to determine if there were any significant differences between sections of college algebra. The analysis revealed that there were no significant differences in any of the traditional sections. There was a statistically significant difference between two of the modeling sections – the one with the highest average score among the modeling sections on all questions and the one with the lowest average score on all questions. The ANOVA did not reveal any differences among the other modeling sections.

	Modeling Approach				Traditional Approach		
	N	ABC	DF		N	ABC	DF
All Questions	217	57.14	42.86		285	41.05	58.95
Skills Questions	251	53.78	46.22		285	35.09	64.91
Modeling Questions	217	61.29	38.71		285	47.02	52.98

Table 5. Percentage of Letter Grades on Final Exam Questions.

Table 5 contains the percentage of A, B, or C grades on all questions as well as the percentages for the skill questions and modeling questions for both approaches to college algebra. These grades were determined by taking the sum of each student's points divided by the total number of points assigned to these questions. They are based on the traditional grading scale ($90 - 100$ is an A, $80 - 89$ is a B, etc.). A two sample test of proportions revealed that when analyzing the data in any of the three ways (all questions, skill questions only, modeling questions only) the proportion of students who completed the modeling-based course and received an A, B, or C on the common exam questions was significantly higher ($p < .01$) than the proportion of students who completed the traditional course and received an A, B, or C on the exam questions. Tables 4 and 5 reveal that when evaluated with identical assessment items, students who studied college algebra through the modeling-based approach performed better on the assessment instrument consisting of modeling and skills questions than students in the traditional sections.

One item worth mentioning which was not determined through a formal assessment is, based on anecdotal data, attendance was significantly better in the modeling sections when compared to the traditional sections. In the traditional sections, the instructors reported a significant drop in attendance after the first $3-4$ weeks of the semester. The instructors of the modeling-based approach said that attendance remained steady throughout the semester with at most $2-3$ students missing on any given day. Based on discussions among instructors, it appears that the modeling-based approach was more successful at getting students to attend class. Undoubtedly, this fact was based on the presence of an attendance policy in the modeling sections and the lack of one in the traditional sections. However, it should be noted that the attendance policy was not adopted by all of the modeling instructors.

Grades in Subsequent Mathematics Courses

College algebra is a prerequisite for two courses at VCU – precalculus and a mathematical applications course for business majors. In Spring 2005, we collected course grades from these two courses on all students who passed college algebra the previous semester. The grade distribution is outlined in Table 6. Sixty-nine percent of the students who passed a modeling-based college algebra course in Fall 2004 took one of these courses and 63% of passing students in a traditional section took one of these courses. For comparison, the table also contains the ABC rate for students who did not take a college algebra course at VCU but placed into precalculus or business mathematics through a placement test or transferred credit for college algebra from another institution.

Based on a two sample test of proportions, the percentage of students passing a subsequent course after completing the modeling-based college algebra course was significantly larger ($p < .001$) than the percentage of students passing precalculus or business mathematics who did not take a college algebra course at VCU. At the 5% level of significance, the modeling-based percentage was not significantly different than the percentage of students passing a subsequent course after completing a traditional section of college algebra. Keep in mind that these percentages are based on grades collected from one semester and some students postpone taking a mathematics course for which college algebra is a prerequisite.

	Modeling Sections (N = 167)	Traditional Sections (N = 399)	Placement or Transfer (N = 294)
ABC	63.47	70.18	44.22
DFW	36.53	29.82	55.78

Table 6. Percentage of Students Passing Subsequent Courses in Spring 2005

Table 7 contains the ABC rates for students who took precalculus in Spring 2005. For students who did not take a college algebra course at VCU, 48.20% received an A, B, or C in precalculus while 56.38% of students who completed the modeling-based college algebra course received a one of these grades in precalculus. Using a two sample test of proportions, these percentages are statistically similar ($p = .11$). In the same semester, 70.42% of students who completed a traditional college algebra course received an A, B, or C in precalculus in the next semester. Using the same statistical test, this percentage is significantly larger ($p < .01$) than the other two percentages. Therefore, a larger percentage of students from the traditional sections received a passing grade in precalculus compared to students who were in a modeling-based section of college algebra or did not take college algebra at VCU.

	Modeling Sections (N = 94)	Traditional Sections (N = 284)	Placement or Transfer (N = 139)
ABC	56.38	70.42	48.20
DFW	43.62	29.58	51.80

Table 7. Percentage of Students Passing Precalculus in Spring 2005.

	Modeling Sections (N = 73)	Traditional Sections (N = 73)	Placement or Transfer (N = 155)
ABC	72.60	69.57	40.65
DFW	27.40	30.43	59.35

Table 8. Percentage of Students Passing Business Mathematics in Spring 2005.

Considering the mathematical applications course for business majors (see Table 8), the percentage of students from a modeling-based section who received an A, B, or C in the business mathematics course is statistically similar ($p = .33$) to the percentage of students who received one of these grades and completed a traditional college algebra course in the previous semester. According to a two sample test of proportions, both of these percentages are statistically larger ($p < .001$) than the 40.65% of students who received an A, B, or C in business mathematics but did not take college algebra at VCU.

The percentages outlined above reveal that students in the modeling-based sections and the traditional sections had similar passing rates in subsequent mathematics courses taken one semester after completing college algebra. The results were most meaningful when the course taken after college algebra was a mathematical applications course for business majors. However, with respect to precalculus, students who completed a traditional college algebra course had a higher passing rate when compared with students who completed the modeling-based college algebra course.

Overall Success in Two Semesters

Percentage Who Passed	Modeling Sections (N = 284)		Traditional Sections (N = 284)
Business Mathematics or Precalculus	37.3		28.3
Precalculus	18.7		20.2
Business Mathematics	18.7		8.1

Table 9. Percentage of Fall 2004 College Algebra Students Who Passed a Subsequent Mathematics Course in Spring 2005.

Table 9 contains percentages that reflect the success of college algebra students over a two semester period: Fall 2004 and Spring 2005. Considering the total number of students enrolled in a section of college algebra in Fall 2004, 37.3% of students who enrolled in a modeling-based section received an A, B, or C in precalculus or business mathematics in Spring 2005 compared to 28.3% of students enrolled in a traditional section received an A, B, or C in one of the subsequent courses. Based on a two sample test of proportions, a significantly larger percentage ($p < .01$) of students who started in a college algebra course with a focus on mathematical modeling passed their next required mathematics course compared with students who started in a traditional section of college algebra. When considering the percentages for the precalculus course alone, 18.7% of students who enrolled in a modeling section in Fall 2004 received an A, B, or C in precalculus in the next semester and 20.2% of students from a traditional section of college algebra received one of these grades. A two sample test of proportions reveals that these percentages are not significantly different ($p = .28$).

With respect to business mathematics, 18.7% of students who first took a modeling-based college algebra course in Fall 2004 then passed business mathematics in Spring 2005 while only 8.1% of students who started in a traditional section of college algebra received a similar grade in business mathematics. The statistical test reveals that the modeling-based percentage is significantly larger ($p < .001$) than the percentage based on success of students in the traditional sections in a subsequent business mathematics course.

Although the students in the modeling-based sections performed better in subsequent courses when using the measure outlined in Table 9, we believe their performance in subsequent courses should be even better. This will be carefully considered as we continue to make changes to our college algebra course. However, there are two factors that are important when considering the performance of college algebra students in subsequent mathematics courses: (1) since 80% of the students were freshmen and 34% of students who passed college algebra in Fall 2004 did not take precalculus or business mathematics in Spring 2005, more students will complete one of these courses in future semesters and (2) some students enrolled in college algebra at VCU are not required to take precalculus or business mathematics.

CONCLUSION

Based on the results of the evaluation of student achievement, the passing rate, and attendance information, we are pleased with the modeling-based approach to college algebra piloted in Fall 2004. We believe these changes have made for a more positive and meaningful experience for students enrolled in the course. Our analysis does not provide details on the influence of specific changes to the college algebra experience of students. Rather, the results of this study are due to the entire collection of curricular and administrative changes made in the modeling-based sections of the course. The positive results are based on a variety of factors including but not limited to (a) the emphasis on mathematical models and applications, (b) the reduction in the amount of time spent on skills, (c) the syllabus (quizzes, projects, attendance policy, etc.), (d) the regular use of group work, (e) the presence of teaching assistants at all class meetings, and (f) the weekly discussions and resulting added attention by modeling-based instructors to the course.

These changes and the others outlined in the description of the approach above resulted in more students completing the course (5.6% withdrew throughout the semester) and a 71.83% passing rate in the course.

The students who completed the modeling course performed better than students in the traditional sections on test questions that evaluated their mathematical skills. These same students performed better than their counterparts in traditional college algebra on a small set of applied problems. Overall, the course analysis reveals that the modeling approach is a positive experience for college algebra students and does not hinder their ability to apply the mathematical skills necessary to successfully use modeling in the evaluation of mathematical problems. When compared with their counterparts in a traditional college algebra course, a statistically similar percentage of students who complete the modeling-based course receive an A, B, or C the next semester in the mathematics course for which college algebra is a prerequisite. Overall a much greater percentage of students who enrolled in a modeling-based section completed a subsequent required mathematics course when compared with the percentage of students in the traditional sections who completed a subsequent course.

One area of concern is the percentage of students who completed a modeling-based course and received an A, B, or C in precalculus. We believe there should be a higher passing rate for students taking precalculus after completing the modeling-based course and will work on finding ways to improve this rate. The data upon which this concern is based is one semester of grades in subsequent courses. We will continue to track the success of students who complete both the modeling and traditional college algebra courses by evaluating the grades of these students in mathematics courses in future semesters.

The development of the ideal college algebra course is an ongoing process at VCU. Our courses are still being revised to meet the needs of the VCU student population. The modeling-based sections in Spring 2005 were taught in a manner similar to what is outlined above with minor changes to the amount of time spent on particular topics to allow for more time to be spent on important mathematical models that are covered at the end of the semester. One difficulty we noted in Fall 2004 with the modeling sections was based on course scheduling. Six of the eight sections met both the 75 minute class period and the 50 minute computer lab session on the same day. These instructors reported that they had a harder time keeping students attention focused on the tasks at hand for 125 minutes of the day. Students felt they were exposed to "too much math" in one day. The other two sections whose lab met on a different day did not have this difficulty. Therefore, for future courses, the modeling-based sections of college algebra will meet at least three days a week with the computer lab session meeting on a different day than the regular class sessions. We are looking closely at the traditional college algebra sections as well. In particular, we are considering instituting an attendance policy and weekly quizzes in the traditional college algebra sections.

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BIOGRAPHICAL SKETCH

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