Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Much Ado About Nothing

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This article is a brief answer to the question: When did zero come into being? The concept and use of zero was a great innovation, easing computation, and is so commonplace that these days we take zero for granted. The present use of zero is easy and obvious, the origins are not.

I use the plural for origins in the previous paragraph, as the use of zero seems to have developed independently on two vastly separated continents sometime in the first millennium. But the origins did not take place where you might think.

The Egyptian civilization left no sign of the use of zero, nor did the Babylonians despite their remarkable achievements in mathematics. The Babylonians used a form of the quadratic equation, and did remarkable work in computing astronomical tables, but their sexagesimal system left no indication of positional notation or symbol for zero. The writings of the Hebrews did not distinguish between 1000 and 1 symbolically. The ancient Greeks used letters for numeration, as did the Romans until the invention of Roman numerals, but the written works of neither civilization indicate the use of a positional zero.

Traditionally, the origins of zero are attributed to the Hindu culture in India, subsequently transmitted to Europe, and to the Orient, from the Arabic world during the Renaissance. An early appearance of zero in India is found in an inscription at Galior in AD 876. Earlier than that, in counting houses in India, a dot was used in writings to represent an empty column on the abacus. That dot was the precursor to the present symbol for zero, therefore the use of a symbol for zero began in India sometime between 300 and 700 AD. Brahmagupta (ca AD 630) knew of the rules for arithmetical operations involving zero, and Moslem writers gave credit to the Hindu origins of the zero sign.

As mentioned previously, it was through the Moslem Arabs that the zero concept reached West-

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ern Europe. An early Latin example of the use of the new system was found in Spain in a manuscript written about 976. It was not until the appearance of Leonardo of Pisa's *Liber Abaci* in 1202 that the use of the Hindu-Arabic system of arithmetic gained a stronghold in Europe. Leonardo (better known as Fibonacci, the originator of the well known Fibonacci sequence) was educated by Islamic teachers, as his father was a prosperous merchant working in North America. Leonardo's text was key to the transmission of the zero concept throughout Europe. There was another place where a symbol for zero was being used, however, that had no direct Hindu connection

Besides having a remarkably accurate calendar, the ancient Mayans of Central America included a positional zero in their computations. The symbol looked similar to a half-closed eye. The Incas of Peru used knots on cords for computation, and the absence of a knot was their way of showing an empty position. So there was an American development of a positional zero, and the innovation occurred sometime between 300 BC and 900 AD.

When did the use of the zero come about? There is no definite answer to that question, although we know that the use of zero started independently in India sometime during the first millennium and, perhaps even earlier, in Central America.

[2] Class Activity: An Impending Star Crash

On Wednesday October 22, 1997, the front page of the New York Times showed images from the Hubble Space Telescope of the fiery collision of two galaxies. The galactic collision, which occurred about 63 million light years away, gave birth to millions of new stars.

Several astronomers are predicting that our own galaxy, the Milky Way, and the galaxy Andromeda are on a collision course. The two galaxies are approaching each other at the speed of 300,000 miles per hour. They are now approximately 2.2 million light years apart (which is approximately 20 times the diameter of the Milky Way).

Questions

a. Use scientific notation to express the number of miles in a light year,

b. What is the approximate diameter of our Milky Way? (Express your answer in terms of miles and in terms of light years.)

c. Assuming that the distance between the Milky Way and Andromeda continues to decrease at the rate of 300,000 miles per hour, how long will it take for the two galaxies to collide?

[3] Small Group Activity: Stopping Time

This is a small group activity. Each group of students should have access to a graphing calculator or a computer algebra system (e.g., Derive, Maple, MathCad, etc.). This activity involves fitting curves to a set of points and interpreting the results.

What is the minimum safe distance that drivers should maintain between vehicles when traveling in the same lane? What factors should be taken into consideration when determining this distance? The stopping distance of a vehicle is the sum of the reaction distance and the braking distance. The reaction distance is the distance the vehicle travels during the reaction time (react, think, and apply brakes). The Texas Drivers Handbook (1995) states that the reaction time is 3/4 second. (Is reaction time dependent on speed? Why?) The Handbook gives the approximate braking distances displayed in the following table (assuming the vehicle has good brakes and tires and is traveling on dry, level, payment).

Your task is to model the stopping distance of a vehicle as a function of speed. That is, develop a function whose input is speed (mph) and whose output is the corresponding stopping distance (feet). The problem is broken down into smaller pieces as described in the following requirements.

Requirement 1. Complete the following table by filling in the Reaction and Stopping columns. The column headings Reaction, Braking, and Stopping refer to distances measured in feet.

Speed(mph)	*Reaction*	*Braking*	*Stopping*
20		23	
30		45	
40		81	
50		133	
60		206	
70		304	

Requirement 2. Determine a reaction function that inputs speed and outputs Reaction distance in feet. Hint: Plot the points, (speed, Reaction distance), and then fit a curve to the points.

Requirement 3. Determine a braking function that inputs speed and outputs braking distance in feet. Hint: Plot the points, (speed, Braking distance), and then fit a curve to the points.

Requirement 4. Determine a stopping function that inputs speed and outputs Stopping distance. Use your function to determine the minimum safe distance between vehicles that are traveling 80 mph.

Requirement 5. The Texas Drivers Handbook (1995) says "A good rule is to stay at least two seconds behind the vehicle ahead." (The "two seconds" refers to the distance you would travel in two seconds.) Develop a two second function that inputs speed and outputs the distance traveled in two seconds. Compare your stopping function model (Requirement 4) and your two second function. Hint: plot the two graphs on the same set of axes and then comment.

Requirement 6. Repeat Requirement 5 replacing The Texas Drivers Handbook rule with the "rule of thumb" that says you should stay at least one car length behind the vehicle ahead for each 10 miles of speed that you are traveling. (That is, if you are traveling 40 mph, then you should have at least 4 car lengths between you and the car ahead.)

Requirement 7. List factors other than speed that a driver should consider in determining a safe distance between vehicles.

[4] Writing Assignment: Tangent Line

Backgound. The two most commonly used temperature scales are the Fahrenheit and Celsius scales. The Fahrenheit scale, developed by the German physicist Gabriel Fahrenheit (1686-1736), is the older of the two scales. The Celsius scale was developed by the Swedish astronomer, Anders Celsius (1704-1744). An important question is "How do you convert Celsius temperature to Fahrenheit temperature?"

Let us form a linear function in which Celsius temperature is the input and Fahrenheit temperature is the output. We know that in the Celsius temperature scale, water freezes at 0^{0} C and boils at 100^{0} C, while in the Fahrenheit temperature scale, water freezes at 32^{0} F and boils at 212^{0} F. To display this data graphically, we plot the Celsius temperature (input variable) on the horizontal axis and the Fahrenheit temperature (output variable) on the vertical axis. The data can then be plotted as two points: the freezing points of the two scales, (0,32), and the boiling points of the two scales, (100,212). Using the slope-point method for determining the equation of a straight line, we define our function as

$$f(c) = \frac{180}{100}c + 32 \quad \text{or} \quad f(c) = 1.8c + 32$$

where c is the temperature measured in the Celsius scale.

Note that the slope (1.8) of the line determined by the two points provides a graphical display of the rate of change of Fahrenheit temperature with respect to Celsius temperature. That is, for every degree change in the Celsius temperature there is a corresponding change of 1.8 degrees in the Fahrenheit temperature.

The purpose of presenting the preceding example was to emphasize that the *slope of a straight line provides a graphical display of the rate of change of the dependent variable with respect to the independent variable.* How can the graphical display of rate of change be generalized to non-linear functions (i.e., functions whose graphs are not straight lines)? For example, how can the parabolic curve representing the path of a ball thrown up into the air, display the rate of change of height with respect to time?

Write a one page essay debating the statement: The slope of the line drawn tangent to the graph of a function at a given point is the rate of change of the function at that point.

[5] Queries

a. What is the slope of the line parallel to the line $2y - 5x = 17$?

b. If you quadruple the area of a circle, how does the circumference change?

c. On a 30 mile trip to the Ft. Worth-Dallas Airport you drive the first 15 miles at 60 mph and for the last 15 miles you drive at 15 mph. What is your average speed for the 30 mile trip?

[6] Notices

• The Joint Mathematics Meetings will be held in Baltimore, Maryland January 7-10, 1998.

a. Professor Laurette Foster, Prairie View A&M University will co-lead the MAA Minicourse #2 "Interdisciplinary Lively Applications." The course meets Friday 8:00-10:00 AM and Saturday 3:15-5:15 PM.

b. The Reunion for participants of past Calculus Reform Workshops will be held Thursday evening 7:00-9:00 PM. Everyone who has attended a Calculus Reform Workshop is invited to attend and bring along a friend. Anyone who would like to give a 10-15 presentation on what they have done as a result of attending a Calculus Reform Workshop is urged to send a short abstract to Don Small, Dept. of Math. Sciences, U.S. Military Academy, West Point, NY 10996 (email ad5712@exmail.usma.edu)

• The *Vision - Potential* Newsletter will not be published in December. The Deadline for contributions to our January Newsletter is

Friday, January 16, 1998.

Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are all welcomed. Please send material to Dr. Della Bell, Chair, Dept. of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

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Abraham Lincoln wrote of himself in his *Short Biography*

> He studied and nearly mastered the six books of Euclid since he was a member of Congress.
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> He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and language. Hence his fondness for Euclid, which he carried with him on the circuit till he could demonstrate with ease all the propositions in the six books.