Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

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Contents

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[1] Handshake Problem

This is a Fun small-group activity that is suitable to use anytime during the semester. (The class is divided into groups of no more than five each.)

Problem: A group of eight people meet and each person shakes hands with each other person exactly once. What is the total number of handshakes?

Allow the groups to "flounder around" while the instructor circulates noting how each of the groups is struggling to develop an approach to the problem. After several minutes, the instructor asks the group (or groups) with the most promising approach to explain their reasoning to the rest of the class. If a promising approach does not seem to be evident, the instructor suggests making a two-column table with one column, say the left one, labeled "Number of People" and the other column labeled "Number of Handshakes." How many handshakes are there among two people? Among three people? The groups then return to their own deliberations. When one or more groups have completed their table, the instructor asks them to explain their method and reasoning for filling out their table. After the explanation, the instructor What-ifs the problem increasing the number of people from eight to ten. The follow-on question is then to develop a formula for the number of handshakes for a group of n people.

If none of the groups are able to fill out their table, the instructor suggests that each group number its members (1, 2, 3, ...). In each group, consider a subgroup of members 1 and 2 and ask how many handshakes are there between these members? Then add member 3 to the subgroup and ask how many handshakes there are among these three members. Repeat the process by adding member 4 and again by adding member 5. Continue this process until a pattern is recognized. (How much assistance the instructor needs to provide depends on how far along in the course the problem is given.)

Here are three ways students might proceed to recognize a pattern from their table data:

a. Plot the data and then fit a curve to the scatter plot. Verify the correctness of the resulting function by comparing its outputs against the values in the table.

b. Recognize the number of handshakes among three people is 1+2, the number of handshakes among four people is 1+2+3, the

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number of handshakes among five people is 1+2+3+4, and so on. This suggests that the number of handshakes among n people is the sum of the first n - 1 positive integers. This offers a good opportunity to relate the often-told story of Gauss as a ten year old school boy.

> His teacher, in an attempt to occupy the class for awhile, set the students to sum the integers from 1 to 100. Almost immediately Gauss turned in his answer of 5,050. He had visualized a column of numbers from 1 to 100 and a second column of decreasing numbers from 100 to 1. Adding across the rows gave him 101 for each row. Thus he had 101 in each of the 100 rows with every number being counted twice. Therefore his answer was $\frac{(101)(100)}{2} = 5,050$.

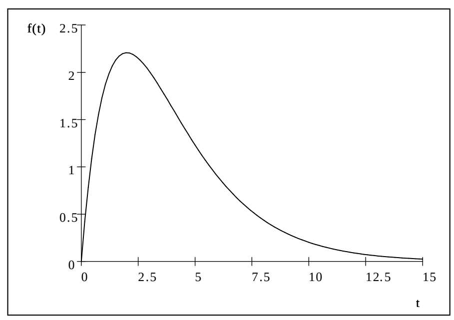
c. Formalize the subgroup process in terms of a recursive sequence. For example when adding a fourth person to a group of three, the number of handshakes is the number among the original three person subgroup plus the handshakes (three) the new person makes with each of the original three members. Abstracting this gives the recursive sequence $h(n) = h(n-1) + n - 1$, where $h(n)$ is the number of handshakes among a group of n people. Although students (probably) do not know how to solve this recursive sequence, they could conjecture a solution by fitting a curve to the data and then substituting the resulting function into the recursive sequence.

[2] Query

Organizations from youth hockey to PTAs to city councils to church choirs often set up telephone calling chains to alert their members to a change. The leader calls designated people, who call their designated people, who call their designated people, and so on. If a calling chain has 121 members, how many calls are necessary to make if each person calls exactly three people?

[3] Surge Functions

Functions of the form $f(t) = ate^{-bt}$ are called surge functions. The following graph of $f(t) = 2te^{-0.5t}$ shows that a surge function initially grows in a nearly linear fashion and then decreases as the negative exponential component dominates.



Surge functions are often used in Pharmacology to model drug concentrations.

The following two examples were derived from the Univ. of Maine Workshop Project Newsletter, *Progressions: Peer-Led Team Learning,* Vol. 7, Issue 4, Summer 2006 (Team members: Paula Drewniany, Sue McGarry, Jen Tyne).

a. Nicotine Concentration. Table 1 gives data on the concentration (mg/dL) of nicotine in a person's blood system t minutes after smoking one cigarette.

Time (min)	Concentration (mg/dL)
0	0
0.1	.85
0.2	1.00
0.3	0.88
0.6	0.35
0.8	0.16
1.0	0.07
1.2	0.03
1.5	0.01

Table 1

Plot this data and then fit a surge function to the scatter plot. What linear function would model the growth in the concentration over [0, 0.1]? What exponential function would model the growth in the concentration over [0.2, 1.2]?

b. Blood Alcohol Concentration (BAC). Data in Table 2 was collected from a group of male drinkers who rapidly consumed two drinks. Time is measured in minutes from the time they first began to drink. The concentration levels are the averages taken over the group. (A concentration level of 27 means that the averages of the men's BAC is .027%.)

Time (minutes)	BAC
0	0
10	27
20	32
30	42
40	43
50	41
60	38
90	26
120	20
150	12
180	9
210	6
240	2

Table 2

Plot this data, showing time (measured in minutes) on the horizontal axis and BAC on the vertical axis and then fit a surge function to the scatter plot. What linear function would model the growth in the concentration over [0, 30]? What exponential function would model the growth in the concentration over [60, 180]?

How much time should elapse from the beginning of the drinking before a member of the group could legally drive?

If the BAC of a member of the group was 38 , could you tell how long it had been since the man began to drink? Explain.

[4] Query

As stated in the proceeding article, the function expression for a surge function is $f(t) = ate^{-bt}$. How does the shape of the graph change when the coefficient a is increased while b is held fixed? How does the shape of the graph change when the coefficient b is increased while a is held fixed?

[5] Class Start-up Challenges

These are small-group in-class activities to begin a class. Groups should be asked to present their solutions along with their reasoning to the class when they have finished a challenge. If all the groups are bogged down, the instructor should lead a discussion in which the different groups talk about what they tried, their thinking, the result, and their reflection on the result. The discussion should gradually lead the thinking toward a promising approach.

a. Let R_1 and R_2 be two numbers. If R_1 is fixed, does $\frac{R_1+R_2}{R_1 R_2}$ increase or decrease as R_2 increases?

b. Arrange the following terms according to increasing magnitude given $0 < x < a < 1$.

$$\frac{1}{a}, \quad \frac{x+a}{a}, \quad \frac{a}{x+a}, \quad Log_{10}(a), \quad \sqrt{a}, \quad a$$

c. Let P and Q be two numbers and let $P > Q > 0$. For each of the following pairs, determine which expression is larger.

i. $P + Q$ or $2P$

ii. $\frac{P}{P+Q}$ or $\frac{P+Q}{2}$

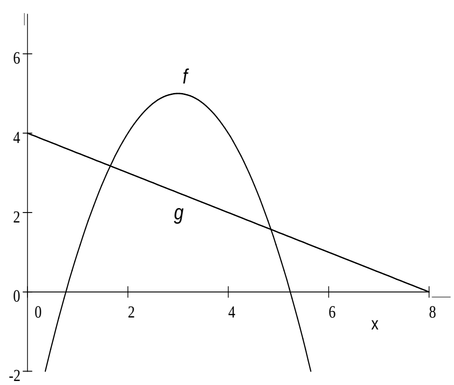
iii. $\frac{Q-P}{2}$ or $\frac{P+Q}{2}$

d. A model for the temperature of ice cream t minutes after being taken out of the freezer is
$\textit{temperature} = a - b * 2^{-t} + b, \quad a > 0, \ b > 0$

i. Is the *temperature* $> a$?

ii. Is the *temperature* $< a + b$?

e. Let the functions f and g be defined by the following multiplot.



Approximate the zeros of $h(x) = f(x) - g(x)$.

Approximate the zeros of $h(x) = f(x) + g(x)$.

[6] Notices

1. Don Small will conduct a workshop on Refocusing College Algebra at the University of Wisconsin, River Falls, April 3-5, 2008. Contact Erick Hofacker (erick.b.hofacker@uwrf.edu) for details.

2. Don Small will conduct a workshop on Refocusing College Algebra at the University of Oregon, April 17-20, 2008. Contact Barbara Edwards (edwards@math.oregonstate.edu) for details.

3. Past issues of the *Vision - Potential* Newsletter are available on our website: www//ContemporaryCollegeAlgebra.org.

4. Deadline for contributions to the September Newsletter is September 10, 2008. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

5. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.

The new 6^{th} edition of

$* * *$ Contemporary College Algebra: Data, Functions, Modeling $* * *$

will be available this June 2008.

“Wisdom is nearer when we stoop, than when we soar.” Wadsworth