Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Activities for Starting Class

The start of a class presents an interesting, and sometimes frustrating, challenge to the instructor. Namely, how to manage the change in the student environment from one of socializing to one of academic learning. Often valuable class time is lost before the students become academically engaged in the lesson. Student centered pedagogy that engages students through small-group activities offers a way to replace the "start-up" challenge with a learning opportunity. The opportunity is to begin the class with a timed, small-group activity that concludes with two or more groups presenting their work to the class. The time limit aspect of the activity encourages students to start working immediately and the presentation aspect provides a means of accountability. In addition, this opportunity addresses the problem-solving and communication goals of the course.

In previous Newsletters, skill exercises were presented as examples of start-up activities. In the following two examples, some mathematical problem solving analysis is required to identify the skill work embedded in word problem settings.

1. A straight playground slide is to be built such that the top of the slide is 4 feet off the ground and the bottom of the slide rests on the ground. In addition the triangular area formed by the slide, the ground, and a vertical ladder to the top of the slide is 10 ft.^{2}.

   a. Determine the equation of the line segment corresponding to a side of the slide.
   b. Determine the slope of the slide.
   c. Interpret the meaning of the vertical intercept of your linear equation from Part a.
   d. Determine the length of the slide.

2. A local Cab Company charges \$1.00 on entering the cab plus \$2.50 for each mile driven.

   a. Develop a fee model that expresses the cost of a trip as a function of the miles driven.
   b. Interpret the meaning of the slope in your function equation in Part a.
   c. Interpret the meaning of the vertical intercept for your function equation.
   d. Plot the fee function from Part a.

* Supported by the U.S. Military Academy.

[2] Painting a Bedroom

Nancy wants to paint the walls of her bedroom which measures 11' by 14' by 8'. The room has two windows that measure (including trim) 40" wide by 64" high, two doorways that are 40" wide by 7' high, and an 8" baseboard that is not to be painted.

How many quarts of paint should Nancy buy? (One quart covers 100 square feet.)

[3] Identifying Functions

(This problem was adapted from one given by Mr. Yarrish at Harrisburg Area Community College) The data in each of the following tables is taken from a linear ($f(x) = mx + b$), quadratic ($f(x) = ax^2 + bx + c$), exponential ($f(x) = a\ 2^x + c$), or common logarithmic ($f(x) = a\log_{10}(x) + c$) functions. For each of the tables, identify the function and determine its equation by determining the values of the parameters.

a. Table 1:

x	-2	- 3	-1	4	2
g(x)	9	11	7	-3	1

b. Table 2:

x	4	1	10	5	8
h(x)	5.2	2	6	5.4	5.8

c. Table 3:

x	-1	0	2	4	-2
k(x)	5.5	8	23	83	4.25

d. Table 4:

x	-1	0	2	5	6
k(x)	-3	2	6	3	-10

[4] Life Insurance Rates

Insure.com listed the following monthly rates for $1,000,000 life insurance policy. (Source: *USA Today*, September 9, 2007.)

Age	Monthly Rate
35	$21
40	$29
45	$47
50	$69
55	$105
60	$157
65	$256

Model the situation:

a. Create a scatter plot for the data.

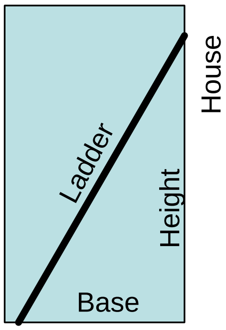
b. Recognize the shape of the scatter plot. Does the shape suggest a quadratic polynomial or an exponential model? Explain.

c. Fit a curve to the scatter plot. (The equation of the curve defines a function which is the model.)

d. Use your model to predict the monthly rate for a person whose age is 70.

[5] Positioning an Extension Ladder

With respect to safety, what is the proper elevation angle for an extension ladder leaning against the side of a house? A brochure of the Lynn Ladder and Scaffolding Company provides the recommendations listed in the following table. The *height* is the vertical distance in feet from the ground to the bearing point of the ladder against the house. The *base distance* is the horizontal distance in feet of the resting point of the ladder from a vertical line passing through the bearing point of the ladder (i.e., horizontal distance of the foot of the ladder from the house).



Height	**Base Distance**
9.5	2.5
13.5	3.5
17.5	4.5
21.5	5.5
25	6.5
28	7
31	8

a. Create a scatter plot of the data. (Which variable is the independent variable?)

b. Fit a curve to the scatter plot.

c. Determine the slope of the ladder using the equation of the curve found in Part b.

d. Determine the angle of elevation of the ladder from the slope in part c.

e. Explain why the answers in parts c and d are reasonable. If the answers are not reasonable, redo your analysis.

[6] Queries

You are interested in designing the largest area that can be enclosed with 100 feet of fencing. For each of the following, be prepared to explain your reasoning and to justify that your answer is in fact a maximum.

a. How large a rectangular area can be enclosed with a 100 foot fence?

b. How large a circular area can be enclosed with a 100 foot fence?

c. How large a triangular area can be enclosed with a 100 foot fence?

[7] Small Group Activity

For this business related activity, divide the class into pairs of students, have them work the following problem, then call on different pairs to describe what they did and to explain their reasoning. Ask how many pairs used a graphical approach? Call on different pairs to present their follow-on problem. This activity could be used to lead into a discussion of the reasonableness of a demand curve being decreasing and a supply curve being increasing (both curves representing functions of price). Another discussion topic emanating from this activity is the interpretation of the intersection point of the supply and demand curves as an equilibrium point. What does equilibrium mean? What happens when the price is changed from the equilibrium price? For example, is the demand higher or lower than the supply when the price is less than the equilibrium price?

Problem. One summer day, Brenda decides to sell apples from her Dad's apple tree to the people walking along her street. She decides that the price of an apple should depend on how many apples she plans to sell. (The more apples she sells, the higher she has to climb in the tree to pick them and thus the amount of her work per apple increases.) She also realizes that as the price per apple increases, fewer people will buy apples. Therefore her supply function is increasing and her demand function is decreasing. Suppose her supply function is $s(p) = p^2/5$ and her demand function is $d(p) = 40 - 2p$ where, in both functions, p denotes the price of an apple. Thus if Brenda thought she could only charge 5 cents per apple, she would just pick 5 apples (the low hanging fruit); while at 15 cents per apple, she would be willing to pick 45 apples. On the demand side Brenda expects that at 5 cents apiece, there is a demand for 30 apples, but at 15 cents per apple the demand is only for 10 apples.

Your tasks are to:

a. Form a multiplot of the supply and demand functions.

b. Interpret the vertical asymptotes of your two plots.

c. Determine the price at which Brenda would "sell out" while satisfying the demand.

d. Determine how much Brenda would make at the price in Part c.

Follow-on Problem. Brenda's grandfather offers to subsidize the price of apples by 5 cents per apple. Assume Brenda adjusts her supply function to reflect her grandfather's subsidy and keeps her original demand function. Now:

e. Form a new multiplot of the supply and demand functions.

f. How does your new multiplot compare with the multiplot in Part a?

g. Determine the price that Brenda would sell out while satisfying the demand.

h. Determine how much Brenda would make at the price in Part g plus her grandfather's subsidy.

i. Develop a different follow-on problem.

[8] Notices

1. The sixth edition of the text *Contemporary College Algebra: Data, Functions, Modeling* is now in preparation. Comments and suggestions for improving the text are welcome. Please send them via e-mail to Don Small <don-small@usma.edu>.

2. The 2008 Joint Mathematics Meetings will be held in San Diego, CA, January 7-9, 2008. Two special sessions of interest are:

   Poster Session: Monday, January 7 from 2:00 to 4:00 PM

   "Sharing a Residue" session, Monday, January 7 from 5:00 to 7:00 PM

3. The next issue of the *Vision-Potential* Newsletter will appear in November 2007. Deadline for contributions to the November Newsletter is Thursday, November 1, 2007. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed. Please send material to Don Small, <don-small@usma.edu>.

4. To subscribe to this Newsletter, write to Don Small, Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996 or e-mail <don-small@usma.edu>.

5. Previous copies of the Vision-Potential newsletter can be found on the website: *contemporarycollegealgebra.org.*

meaningful experiences resulting in personal growth for all students in terms of helping them become confident and competent problem solvers. Our objective must be to help prepare students to think clearly and deeply about quantitative issues in a world that is becoming more complex and more uncertain. Although developing skills in "problem solving in the modeling sense" is difficult to teach and even more difficult to assess, these are the essential life skills our students need.

Helping students *learn how to learn* and helping them develop *habits of mind* for learning, need to be primary concerns in the preparation of every lesson. Some examples of skills that address these concerns include:

a. Sketch a picture to illustrate the situation.
b. Iterate the paradigm " TRY something (e.g., guess), note the errors, modify the approach to reduce the errors, and try again" until an acceptable conclusion is obtained.
c. Ask: Does my answer make sense in terms of the original setting?
d. "What-if" an exercise to obtain a conceptual understanding of the problem underlying the exercise.
e. Look for examples of similar situations in other settings.

The Contemporary College Algebra program is very much in sync with the recommendations of The Mathematical Association of America (MAA). A recent Report on Introductory Courses to the MAA's Committee on the Undergraduate program in Mathematics states "A clear and dominant focus of all of these courses should include the following goals:"

a. *Create confident and competent problem solvers[3]*
b. *Emphasize learning with understanding*
c. *Support the actual mathematical needs of other disciplines*
d. *Provide life-time relevance to students*
e. *Employ student-centered pedagogy*