Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

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[1] Modeling in College Algebra

Modeling in (refocused) college algebra programs is, in part, a response to the increased emphasis being placed on quantitative analysis in most disciplines. Today's technology in the form of graphing calculators and spread sheets provide college algebra students with tools that enable them to model situations that in the past were only found in upper-level courses. There are huge advantages to doing this - it far better connects the mathematical ideas to the kinds of applications that students will see in their other quantitative courses; it gives the students a much better sense for the usefulness of the mathematics; it allows the instructor to return to and reinforce the properties (both behavioral and manipulative) of the different families of functions; and it serves as a unifying theme in the course (something which is sadly lacking in traditional courses at this level).

* Supported by the U.S. Military Academy.

In general, there are three levels of modeling found in refocused college algebra courses. The beginning level consists of plotting functions and analyzing the graphs. Developing a sense of the shapes of the basic function categories is an important outcome of this level of modeling. The second level involves gathering data (or being presented with it), plotting it, fitting a curve to the scatter plot, and then using the resulting function to inform the original situation. Graphically fitting a curve to a scatter plot makes use of the knowledge of the shapes of the basic function categories from level one. Graphically fitting curves to scatter plots raises the question of "best fit" which leads into working with the sum of squared errors and regression. The third level involves modeling with recursive sequences. This topic serves to provide a feel for the power of mathematics in modeling the real world scenarios; it serves to reinforce the behavioral characteristics of various categories of functions; it links the mathematics to what is done in many of the other disciplines; and it provides students with the knowledge and understanding of recursion - the mathematical language of the spreadsheet, which is the primary technological tool of almost every other quantitative discipline.

Aspects of the first level of modeling are found in traditional college algebra courses.

The objective in these courses, however, is usually just to plot a function, not to develop a base on which to move into level two or level three. The modeling objective in refocused college algebra courses is to provide students with a basis on which to address the quantitative situations they will encounter in other courses as well as providing them with an appreciation of the applicability of mathematics.

[2] Fundamental Skills
(Elementary Algebra)

Suggestion for a quick class activity: Pair off the students and give each pair one of the questions on the following list or a similar one that you make up. After three minutes, ask each pair to present and explain their answer. (Additional lists of fundamental skill questions can be found in the January and February Newsletters.)

For each of the following equations, state if it is True or False and then give an example that supports your answer. (That is, assign values to each of the variables and then compute the value of each side of the equation.)

a. $(x^2+3)^2-6-2x^2=(x^2+3)(x^2-1)$

b. $(a-b)^2=a^2+b^2$

c. $a(b^3-b^{-3})=0$

d. $\sqrt{a^2+2ab+b^2}=a+b$

e. $\frac{a}{x+b}=\frac{a}{x}+\frac{a}{b}$

f. $a-b(x-1)=a-bx-b$

g. $\frac{a+bx}{a}=1+bx$

[3] Modeling the Amount of Gas in a Gas Tank

Zeph's car has an 18 gallon gas tank and gets 24 miles per gallon on average. Develop a model for the amount of gas in the gas tank as a function of the number of miles driven since the last fill-up.

What is the input or independent variable?

What is the output or dependent variable?

What is the domain of your function? What is the range of your function?

Considering the context (driving, amount of gas), should your function be increasing or decreasing? Is it?

Plot your function.

Give a physical interpretation of the slope of the graph.

[4] Coffee and Caffeine

(This problem illustrates the need to iterate the modeling process in order to obtain a reasonably accurate model.)

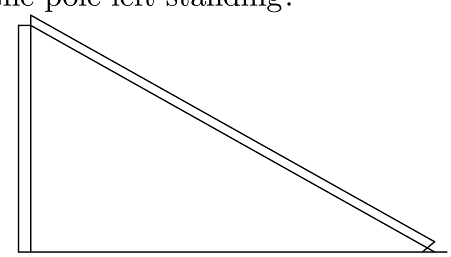
Tony likes his coffee strong and black at 90 mg. of caffeine per 8 ounce cup (no decaf for Tony) and is known to drink six or more cups per day. Lately, however, he has become concerned over his growing restlessness, nervousness, and insomnia that he attributes to the build up of caffeine in his body. He decides to limit himself to three cups of coffee per day, one at 7:00 am, one at noon, and one at 6:00 pm. Tony has heard that his kidneys will filter out 13% of the caffeine in his body every hour. He wonders how many milligrams of caffeine from coffee will be in his system at ten o'clock at night when he goes to bed if he stays with his three cups per day. Please show Tony how to model the amount of coffee caffeine in his system.

Hint. To get started with your model, you might assume that the initial amount of coffee caffeine in Tony's system when he begins to drink his morning coffee is zero. Then determine the amount of coffee caffeine in his system after he drinks his noon cup and then again after he drinks his evening cup. Now extend your model to calculate the amount of coffee caffeine in Tony's system just before he has his morning coffee the following day. If this value is not zero, then you should recalculate using this value for the initial value (in place of zero). You may need to iterate this process a few times in order to gain a reasonable approximation to the initial value.

[5] Quickies

The following questions were taken from the Texas TAKS exam.

a. A wooden pole was broken during a windstorm. Before it broke, the total height of the pole above ground was 25 feet. After it broke, it looked like the following picture with the top of the pole touching the ground 15 feet from the original base. How tall is the part of the pole left standing?



b. Doris decided to change her circular flower garden for a square garden. Her circular garden, which was 30 feet in radius, had a fence around it. She wants to use the fence to enclose her new garden. What is the maximum size that her new garden can be? Will this garden have more or less area than her circular garden?

c. Marshia brought cookies to school. She gave a third of her cookies to Ana. Ana then gave a fourth of her cookies to Cybil. Cybil gave half of her cookies to Betsy. If Betsy received two cookies, how many cookies did Marshia bring to school?

[6] Push-ups

When Sam realized that he was physically out of shape and needed to adopt a physical therapy program, he decided to include doing pushups every morning. To track his progress, he recorded the number of pushups he did on Monday mornings over a ten week period. His record of correctly executed push-ups is:

Week	# Push-ups
1	20
2	28
3	35
4	40
5	44
6	47
7	49
8	52
9	54
10	55

a. Develop a model based on a scatter plot of this data.

b. How "good" is your model? Explain.

c. If Sam continued his regime of doing push-ups, how many push-ups does your model indicate that Sam will do at the beginning of week 12? Does this make sense? If not, what can you say about your model?

(This problem could be made more relevant to your class by generating your own data. You could ask some of the athletes in your class, who are in training, to volunteer to do push-ups every morning and to record their data over a ten day period.)

[7] Two Modeling Problems

a. Blood pressure is the force exerted by the blood on blood vessels. There are two standard types of pressure: Systolic-pressure against the artery walls when the heart has just finished pumping out blood and Diastolic-pressure against the artery walls between heartbeats, when the heart is relaxed and filling with blood. It is common for healthy individuals to have systolic pressures of about 120 millimeters of mercury (mmHg) and diastolic pressures of 80 mmHg during periods of rest. Blood pressure is typically recorded as 120/80, read "120 over 80." Determine a function that models the pressure of blood on a person's arteries as a function of time.

b. There are approximately 152,000,000 working members of the United States population. Every year, between January 1 and April 15, these people file income tax forms with the Internal Revenue Service (IRS) The IRS comments that the number of weekly submissions increases as the April 15 deadline nears. Determine a function to model the number of outstanding submissions as a function of time. Sketch the graph of your function and explain your reasoning.

[8] Zeros of Functions

A zero of a function, f, is a number for which the function value is zero. Thus 2 is a zero of $f(x) = x - 2$. The zeros of a function can also be described as the values of x for which the graph of the function touches the x-axis.

a. Determine the zeros of $f(x) = x^3 - x^2 - 6x$ by plotting the function.

b. Consider a function f with domain [-2,8] that has a zero of multiplicity two (i.e., a double root) at $x = 5$ and a single zero at $x = 0$.

i. Sketch the graph of a function with these properties.
ii. Give an equation that defines a function g with these properties.
iii. Does the function $h = -g$ also satisfy these properties?
iv. Give an equation different from the one in part ii. that defines a function 'satisfying these properties.

[9] Notices

1. The Retreat for the 2007 cohort of schools under the HBCU Retreat and Follow-On program will be held at the U.S. Military Academy, June 4-7, 2007. Contact Don Small (don-small@usma.edu) for information and application forms for the two year HBCU Retreat and Follow-On program.

2. MAA PREP Workshop: *Revitalizing College Algebra*, June 18-21, 2007 at the University of Arizona. Facilitators are Don Small, Bill McCullum, and Bill Haver. The workshop is co-sponsored by the MAA and the Institute for Mathematics and Education (Univ. of Arizona). Registration Fee: \$300 by May 7, \$400 after May 7. (Registration fee, before May 7, is reduced to \$200 for the second, third, and fourth participants from the same department.)

3. <u>Deadline</u> for contributions to the April Newsletter is Monday, April 2, 2007. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

4. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.