Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Developing Mathematical Intuition

There is no "Book" on how to develop mathematical intuition. In fact, the phrase *mathematical intuition* is probably not well defined. Nevertheless, most mathematicians would probably say that their mathematical intuition is an important aspect in their understanding and ability to do mathematics. This suggests that college algebra courses should address the challenge of developing student's mathematical intuition. What can be done?

Experience doing mathematics is certainly a critical component in developing one's mathematical maturity. Thus given a one semester course that is probably the terminal mathematics course for most of our students, we are faced with the question: What types of student experiences should we emphasize in order to maximize the development of math ematical intuition? Is the *drill and skill* emphasized in the past best? Or, is it best to emphasis *modeling* experiences? Or, is it best to emphasize *discovery* experiences? Or, is it best to . . . . ?

* Supported by the U.S. Military Academy.

Many of the experiences designed to help students develop into exploratory learners also serve to develop students' mathematical intuitions. Here are a few queries whose discussions could (hopefully) enhance a student's mathematical intuition.

a. Which of the following choices : (i) 2 gallons, (ii) 4 gallons, (iii) 6 gallons, (iv) 8 gallons is the best approximation of the amount of water in one cubic foot? (Hint: Think of how many half gallon milk cartons are needed to form a cubic foot block.)

b. Is there a relationship between the length of the boundary of an enclosed two-dimensional region and the area of the enclosed region? Hint: Experiment with a circle or triangle or rectangle.

c. What does the highway sign *6% grade* indicate?

d. What would be the angle of elevation of a road if its grade was 100%?

e. What, if anything, is common to the meanings of the terms pitch (as used by a roofing carpenter), grade (as used by a highway engineer), "slope" (as used by a skier), slant (as used with a slide in playground equipment), ramp incline (as used by movers)?

[2] Fundamental Skills
(Elementary Algebra)

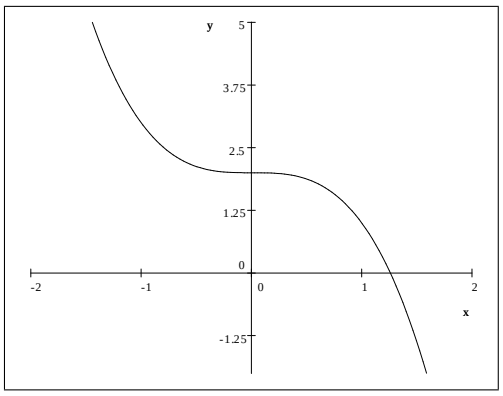
(Please refer to the article on Fundamental Skills in the January issue of this Newsletter.)
This is short activity to begin a Monday morning class. Divide the class into groups of two and then have each group do the following: For each of the five equations, state if it is True or False and then give an example that supports your answer. (That is, assign integer values to "a", "b", "x" and then compute the value of each side of the equation.) After a few minutes call on a group to explain what they did for equation a. Repeat for each of the other four equations.)
a. $(-a)^2 = a^2$
b. $2x^3 + 4x - 6x^2 = 2x(x^2 - 3x + 2)$
c. $(a - b)^2 = a^2 - b^2$
d. $\sqrt{a^2 + b^2} = a + b$
e. $\frac{1}{x+2} - \frac{1}{x} = \frac{-2}{x^2+2}$

Readers are invited to send their lists of fundamental skills to this Newsletter along with suggestions for teaching fundamental skills.

[3] Class Quickies

The following four Class Quickies are examples of short (quick) exercises that can be given to begin a class on a participatory mode or to reengage students who have or are drifting off.

a. Which of the following functions best models the graph?



$f(x) = (x + 2)^2 - 3$
$f(x) = (x - 2)^3$
$f(x) = (-x)^3 + 2$
$f(x) = x^2 - 2$

b. Sketch the graph of $h(t) = 1 + \frac{2}{t^2}$ and give the coordinates of three points on your graph.

c. A line is drawn through the points (-7,2) and (5,8) and another line is described by the function $f(x) = \frac{1}{2}(x + 657)$. What is the relationship between the two lines?

d. This weekend you plan to spend several hours studying. If you plan to spend 0.15 of your time reading an English play, $\frac{1}{4}$ of your time studying math, 20% of your time studying chemistry, and 3 hours studying other subjects and reviewing, how much time will you spend all together?

[4] Temperature Distribution in a Rod

(Instructor: This in-class activity involves modeling with a system of equations. Solving the system offers opportunities to explore different approaches. The follow-on questions offer nice opportunities for students to explain their reasoning to the class.)

Consider five points A, B, C, D, E ordered from left to right on an unevenly heated rod. Let points A and E be the endpoints of the rod and points B, C, and D be interior points. The temperature at point A is maintained at 0 degrees (lowest temperature on the rod) and the temperature at point E is maintained at 40 degrees (the highest temperature on the rod). Assume for each interior point that the temperature is the average of the temperatures at their adjacent points. (Note that A and C are adjacent points to B.) Model the temperature distribution with a system of equations and then solve for the temperatures at points B, C, and D.

Follow-on Questions:

1. Does the location of the interior points make a difference? Explain your reasoning.

2. Sketch the graph of a temperature function for the rod assuming that the points B, C, and D are clustered near point A.

3. Sketch the graph of a temperature function for the rod assuming that the points B, C, and D are clustered near point E.

4. Sketch the graph of a temperature function for the rod assuming that the points B, C, and D are evenly distributed between points A and E.

[5] *PREP* Workshop -
Revitalizing College Algebra

(*PREP* is an acronym for PRofessional Enhancement Programs of the Mathematical Association of America (MAA). The programs are sponsored by the MAA and funded through a grant from the National Science Foundation.)

Revitalizing College Algebra

Bill McCallum, Don Small, Bill Haver
June 18 - 21, 2007
University of Arizona
Tucson, AZ
Co-sponsored by the Institute for Mathematics and Education
Registration Fee: $300 by May 7, $400 after May 7.

Priority will be given to departments that send at least two participants. The registration fee will be reduced to $200 for the second, third, and fourth participant from a department registering by May 7.

A growing number of colleges and universities are modifying their college algebra courses to focus on mathematical modeling. Students address problems presented as real world situations by creating and interpreting mathematical models. Solutions to the problems are formulated, validated, and analyzed using mental, paper-and-pencil, algebraic, and technology-based techniques as appropriate. MAA's Committee on Curriculum Renewal Across the First Two Years (CRAFTY) recommends that all College Algebra courses be organized in this manner. The recently approved CRAFTY College Algebra Guidelines describe the features of such a course. Complementing the modeling approach is an approach aimed at making symbols more meaningful for students by emphasizing structure and purpose in expressions, formulas and equations.

The workshop is designed to help faculty from departments who have plans to renew their algebra curriculum. During the course of the workshop, participants will:

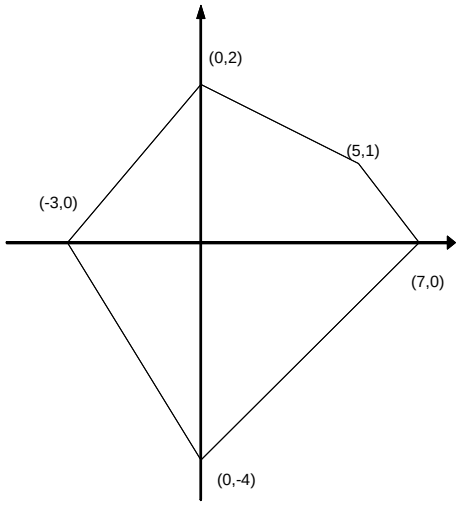
- Explore a number of College Algebra topics from a modeling based perspective;
- Explore problems that help students interpret the structure of symbolic representations;
- Learn about a number of possible texts;
- Consider testing and assessment issuers by reviewing tests and assignments that have been used at other institutions;
- Review group projects that have been assigned to students
- Consider issues of training part time instructors or graduate students who have not experienced courses of this nature;
- Discuss challenges involved in refocusing college algebra courses.

Participants will be provided with some reading before the workshop and will be asked to share their experiences and provide mutual support as they revise their college algebra courses.

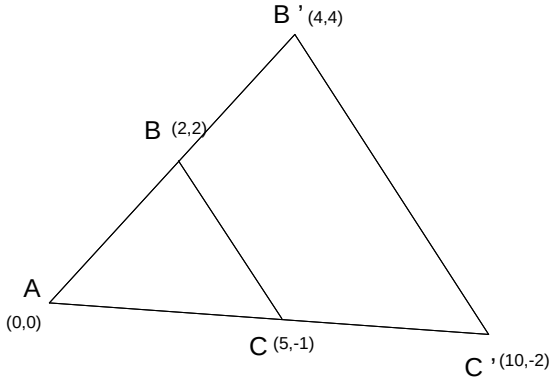
For more information on the Institute for Mathematics and Education, visit http://ime.math.arizona.edu.

[6] Two Geometrical Problems

a. Determine the area enclosed by the polygon pictured below.



b. Dilation. Informally dilations are projections of figures in the plane from the perspective of the origin of a coordinate system. Formally, a dilation centered at the origin with scale factor r maps the point (x, y) to the point (rx, ry). For example, a dilation centered at the origin with scale factor 2 would map the triangle ABC to triangle AB'C'.



How does a dilation centered at the origin with scale factor r affect the area of a triangle? In particular, what is the relation between the area of triangle ABC and triangle AB'C' in the preceding example?

If a dilation centered at the origin with scale factor r is applied to a triangle of area 10, what is the area of the resulting triangle? Does it matter if all of the vertices of the original triangle are different from the origin? Explain your reasoning.

[7] Notices

1. The Retreat for the 2007 cohort of schools under the HBCU Retreat and Follow-On program will be held at the U.S. Military Academy, June 4-7, 2007. Contact Don Small (don-small@usma.edu) for information and application forms for the two year HBCU Retreat and Follow-On program.

2. Deadline for contributions to the March Newsletter is Thursday, March 1, 2007. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

3. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.