Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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Contents

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[1] Equations Then, Functions Now

Since the beginning of the college algebra era (approximately 1960), the primary emphasis in college algebra has been on solving equations. Thus factoring, rules of signs, completing the square, etc. were important techniques and a great deal of time was spent on trying to get students to master them. The choice of equations (mostly first, second, and third degree polynomials) was limited to those that could be solved by the "standard" techniques. Thus a student would probably not be asked to solve the simple looking equation: $y = x^4 - x^2 - 4$. A ramification of this emphasis on solving equations is that students do not generally learn to distinguish between equations and functions. Another ramification is that students do not *recognize* examples of college algebra in the "real world" because very few equations appear in the popular press or in business publications or on news broadcasts. This has contributed to the popular sense that college algebra is an "abstract fog (bog)" only a few can decipher.

* Supported by the U.S. Military Academy

The reform movements over the past fifteen years, aided by technology, have stessed the multiple representation of functions-graphically, numerically, symbolically, and in written form. With a graphing device (calculator, computer), solving equations no longer requires mastering the techniques of traditional college algebra. Furthermore, there are essentially no limitations placed on the type of equation to be solved. Function values can be easily displayed by tables and iterations. Students see functions in graphical and symbolic form throughout the popular press, in business publications, and on news broadcasts.

Today, in refocused college algebra courses, the emphasis has changed from equations to functions. In particular, a major objective of these courses is to have students experience developing functions through the modeling process. For example, students collect data on vehicle breaking distance, plot the data, fit a curve (function graph) to the scatter plot, and then use the resulting function for predictive purposes. Another example is the use of recursive sequences (functions) to model discrete change (i.e., credit car payments) or to approximate continuous change (e.g., warming a cold can of soda). There are huge advantages to this modeling - it helps connect the mathematical ideas with the kinds of experiences students have in their everyday lives; it gives the students a strong sense for the usefulness of the mathematics; it serves to reinforce the behaverial characteristics of various families of functions; and it serves as a unifying theme in the course. Furthermore it provides students with the knowledge and understanding of recursion - the mathematical language of the spreadsheet, which is the primary technology tool of almost every other discipline. In addition, it is fun!

[2] Fundamental Skills (Elementary Algebra)

Developing and maintaining skill in applying elementary algebra operations is important for several reasons including building self-confidence, easing computations, and developing mathematical insight. Like most other skills, repeated usage is necessary to maintain proficency. Because the lists of elementary algebra skills expected of students often differs from instructor to instructor, instructors are urged to talk with each other in order to form a consensus. A next step is to provide students with frequent opportunities to check their skills (e.g., five minute opening class activity, five minute quiz, etc.). The following set of five questions is an example of an "opening class" activity.

For each of the following, state if it is True or False and then give an example that supports your answer. (That is, assign interger values to "a" and "b" and then compute the value of each side of the equation.)

a. $(-2a^2)^3 = 2a^6$

b. $(\frac{2}{a})^{-2} = \frac{a^2}{4}$

c. $ab + 3a^2 - 4a = a(b + 3a - 4)$

d. $\frac{1}{a+2} - a = \frac{-2}{a+2}$

e. $(\frac{1}{a} - \frac{1}{b})^2 = \frac{(b-a)^2}{ab}$

Readers are invited to send their lists of fundamental skills to this Newsletter along with suggestions for teaching fundamental skills.

[3] Class Activity: Generalizing the Concepts of Mean and Median to Two Dimensions

The mean (average) of a one-dimensional, numerical data set is the number computed by adding all the data values together and dividing the resulting sum by the number of data entries.

The median of a one-dimensional, numerical data set is the middle number when the data is arranged in numerical order. If the set contains an even number of entries, then the median is the average of the two center most elements.

A fascinating question is how to extend these two concepts to a two-dimensional set? In particular, how do you determine the mean and median of the population of the conterminous United States (48 states plus the District of Columbia)? The Census Bureau describes the Mean and Median Centers as follows:

Mean center of population as: "The point at which an imaginary, flat, weightless, and rigid map of the United States would balance if weights of identical value were placed on it so that each weight represented the location of one person."

Median center of population as "The intersection of two median lines, a north-south line (a meridian of longitude) constructed so that half of the Nation's population lives east and half lives west of it, and an east-west line (a parallel of latitude) selected so that half of the Nation's population lives north and half lives south of it."

The following tables give the Mean Center and the Median Center of the U.S. population for selected years. (Source www. census.gov/population/censusdata/popctr.pdf)

Year	North Latitude	West Longitude
1990	37°52′20″	91°12′55″
1970	38°27′47″	89°42′22″
1940	38°56′54″	87°22′35″
1910	39°10′12″	86°32′20″
1880	39°04′08″	84°39′40″
1850	38°59′00″	81°19′00″
1820	39°05′42″	78°33′00″
1790	39°16′30″	76°11′12″

Table A: Mean Center

Year	North Latitude	West Longitude
1990	38°57′55″	86°31′53″
1970	39°47′43″	85°31′43″
1940	40°04′18″	84°40′11″
1910	40°07′33″	85°02′00″
1880	39°57′00″	84°07′12″

Table B: Median Center

Activity

a. Using the data in Tables A and B, draw a line plot of the Mean Center and Median Center on a map of the United States that shows latitude and longitude lines.

b. Write a paper interpreting the data and your line plots. Include (but do not be limited to) comments on: the overall trend; reasons for the overall trend; predictions for 2000 and 2010.

c. True or False: Is it possible for the Mean and Median Centers to move in opposite directions going north and south or going east and west. State your reasons. If it is possible for the centers to move in opposite directions, explain a scenario where this could happen.

d. Describe how you would determine the *Geographic Center* of the conterminous United States.

e. Explain the justification of the use of the cosine function in the following description for measuring longitude: For these distances, a degree of longitude at the equator was the unit of measurement. East-west distances along the equator could be measured in degrees, but any east-west degree distance north of the equator – where all the United States is located – had to be adjusted to recognize the convergence of meridians toward the poles. This adjustment required that each east-west distance, stated in degrees of longitude, be multiplied by the cosine of the latitude. This mathematical relationship is precise for a sphere and a very close approximation for the earth. (Source: www.census.gov/geo/www /cenpop/calculate)

Follow-on class activity: students determine the Median Center for their class.

[4] Television Sets

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The following chart represents the number of television sets per 1000 people in the Developed Countries between 1985 and 2003.

Year	Television Sets per 1000 people
1985	444.4
1987	460.4
1990	475.9
1995	525.4
2000	600.4
2002	626

(Source: International Telecommunication Union and Earth Trends)

Plot the points on your calculator, where x represents the number of years after 1985.

a. Find the equation of the line that passes through the points representing the years 1990 and 2002. Clearly show your work.

b. What does the slope of the linear model found in (a) represent here?

c. What is the value of the y-intercept of your linear model? And what does it represent?

d. According to the model found in (a), what should the production level have been in 1999? Show your work.

e. According to the model found in (a) and using algebra, determine in which year there were 300 television sets per 1000 people. Show your work.

[5] Notices

1. Applications for the HBCU Retreat and Follow-On program are now being accepted. The goal of the program is to assist schools in refocusing their college algebra courses. Five HBCUs will be selected to send a 3-person team each to a four-day Retreat at the U.S. Military Academy next June. The participants will experience engaging in a refocused college algebra course and each team will draft a syllabus for a refocused course at their school. They will "polish" their draft during the summer and then implement it in one or more pilot sections next fall. Each school will be assigned a mentor who will make two on-site visits to their school next year. Also each school will be encouraged to apply for a $5,000 mini-grant to support their new program. The program, funded by the National Science Foundation, will cover all participant expenses. To apply for the program, contact Don Small at don-small@usma.edu or call (845) 938-2227.

2. The MAA will sponsor a PREP Workshop on Refocusing College Algebra at the University of Arizona June 18-21, 2007.

3. A panel session on *Refocusing College Algebra* will be held Monday morning at 9:00 am on January 8, 2007 as part of the Joint Mathematics Meeting in New Orleans. The panelists will be representatives of the six HBCU schools participating in the NSF funded HBCU Retreat and Follow On program. The panelists will discuss their experiences in refocusing their college algebra courses.

4. Laurette Foster and Don Small will present a minicourse at the Joint Mathematics Meetings in New Orleans, January 5-8, 2007. The title of the minicourse is *Contemporary College Algebra: A Refocused College Algebra Course.* Part A will be offered on Friday, 2:15 to 4:15 pm and Part B will be offered on Sunday, 3:30 to 5:30 pm.

5. Deadline for contributions to the February Newsletter is Thursday, February 1, 2007. Opinion articles, suggestions for writing assignments, small group inclass activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

6. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via email at don-small@usma.edu.