

Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

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[1] Refocusing College Algebra

The national movement to refocus college algebra from being a pre-precalculus course to one that provides a basis for addressing the quantitative needs students will encounter in society, the work place, and in their other courses is rapidly expanding. Within the past few years all of the major mathematical associations have published curriculum guidelines that contain specific recommendations for refocusing college algebra. These guidelines call for introducing elementary data analysis, modeling, use of technology, increased emphasis on communication and on small group work. With respect to pedagogy, the recommendations call for a change from instructor centered to student centered instruction.

The modeling theme in refocused courses requires a deeper understanding of functions than is usually found in traditional courses. Many small group activities involve using

* Supported by the U.S. Military Academy.

graphing calculators or spreadsheets to fit functions to scatter plots. Success in these projects require students to be able to recognize the shapes of the basic functions and to understand their properties. Modeling with recursive sequences (difference equations, discrete dynamical systems) is another activity that distinguishes refocused courses from the traditional courses. These activities provide students with a feel for the power of mathematics in modeling the real world; it serves to reinforce the behavioral characteristics of various families of functions; it links the mathematics to what is done in many of the other disciplines; and it provides the students with the knowledge and understanding of recursion - the mathematical language of the spreadsheet, which is the primary technological tool of almost every quantitative discipline.

The encouragement and support for refocusing college algebra has expanded beyond curriculum guidelines. Consider the following:

(1) Next month (November 2006) the Mathematical Association of America (MAA) will hold a small working conference on "Algebra: Gateway to a Technological Future;" (2) In 2007, the Center for the Study of Mathematics Curriculums is planning a national conference on algebra, one portion of which will be on college algebra; (3) The American Association of State Colleges and Universities is developing an initiative to focus on success in mathematics which will include an empha-

sis on college algebra; (4) The Mayor’s office of the City of San Antonio has established a government/industry/academic task force to improve the college algebra courses and increase student success in them; (5) The National Science Foundation (NSF) is funding an MAA program to renew college algebra at eleven colleges and universities; (6) The NSF is funding a three year program at the U.S. Military Academy to assist eleven Historically Black Colleges and Universities to refocus their college algebra programs.

Leaders in the Contemporary College Algebra program have been instrumental in deepening the national movement to refocus college algebra.

[2] Graphing “Sense”

Graphing programs on calculators or computers offer wonderful opportunities for graphical exploration. However, they also present a danger as they allow students to plot with very little “thinking input.” As an instructor whose teaching career began long before the development of graphing calculators, I am impressed with the ability to show the effects of changing a parameter by just moving a slider bar. However, many of my students see it as just a game. Helping students develop a graphing sense involves hand sketching and written explanations of the effects of changing parameter values as well as using technology for explorations. The following activity is directed toward the hand sketching aspect of graphing.

In each exercise, students working in pairs are asked to sketch (without technology) a multi-plot of the two given functions and then compare the two functions taking into account: domain, long-term behavior, dominance of one function over the other, intersections, asymptotes, etc.

a. $f(x) = \frac{1}{x}$, $g(x) = \frac{1}{x^2}$

b. $f(x) = 2^x$, $g(x) = 3^x$
 c. $f(x) = \sin(x)$, $g(x) = \sin(2x)$
 d. $f(x) = 2^{-x}$, $g(x) = \frac{1}{x}$
 e. $f(x) = x^{1/4}$, $g(x) = \log(x)$
 f. $f(x) = \log(x)$, $g(x) = \log(2x)$
 g. $f(x) = 0.8^x$, $g(x) = 0.5^x$

[3] Algebraic Manipulations

Several students have difficulty with “basic” algebraic manipulations even though they may have spent considerable time “drilling” on them. These students would be better served if part of their drill time was replaced with activities directed toward conceptual understanding. Having students analyze expressions for correctness, as the following illustrates, is one such possibility.

For each of the following, state if it is True or False and then give two numerical examples that support your answer. (For instance, assign integer values to “ a ” and “ b ” and then compute the value of each side of the equation.)

a. $-a^2 = a^2$
 b. $(-a)^2 = a^2$
 c. $(a - b)^2 = a^2 - b^2$
 d. $\sqrt{a^2 + b^2} = a + b$
 e. $\frac{a+b}{1/a+1/b} = ab$

[4] Three In-class Activities

1. Paper Folding. When a single sheet of paper is folded in half, the result is two *sheets*. A second folding of the two *sheets* produces four *sheets*. Make a table with three columns - the first column shows the number of folds from 0 to 5, the second column shows the number of *sheets*, and the third column expresses the number of *sheets* in exponential form. Model this folding phenomenon by expressing the number of *sheets* as a function of the number of folds. Now create a thickness function by multiplying your *sheet* function

by the thickness of one *sheet*. (How can you determine the thickness of a *sheet* since a single *sheet* is too thin to measure accurately?)

- a. Estimate the thickness after you have made 10 folds? 20 folds? 50 folds?
- b. Compute the thickness in feet after 10 folds, 20 folds, and then compute the thickness in miles for 50 folds.
- c. How many (theoretical) folds would be required for the stack of *sheets* to reach the moon, the sun?

2. Sailboat Sail. A Marine Sail company makes and sells sails at a cost of \$30 per square foot of finished sail. How much would a triangular sail with dimensions 33 feet by 27 feet by 21 feet cost? (Does the sail form a right triangle? How do you know?)

Hints

- a. Draw a sketch of the sail and denote the lengths of the sides of the triangle.
- b. Expand your sketch by enclosing it in a right triangle. Label the unknown portions of the sides with variables.
- c. Use an inverse cosine function to determine the size of an angle.

3. Assume the Newton dormitory on your campus has three floors and is approximately 70 feet tall whereas the Euler dormitory has nine floors and is approximately 170 feet tall. Both buildings have flat roofs and the height measurements include the basements. Develop a model that predicts the height of a dormitory building as a function of the number of floors it contains.

[5] Shopping for Lower Gasoline Prices

The volatility of gasoline prices often means that prices across a region are not always uniform. It is not unusual for gas stations within ten miles of each other to charge different amounts for the same type of gas. The variation in price encourages some people to “shop around” to get the lowest price. How

does one determine if traveling to a station for a lower price actually saves money? For example, suppose Sam gets 24 mpg with his car and usually buys 12 gallons when he “fills up.” Also suppose that Sam lives near a gas station that charges \$2.70 per gallon and he knows of another station 10 miles away that charges \$2.60 per gallon. Ignoring the time involved and the wear on the car, would Sam save money buying gas at the station 10 miles away? Explain your reasoning.

Follow up question: Ignoring time and wear on the car, how far could Sam drive to buy gas at \$2.55 and still save money?

[6] Cardiovascular Fitness

(This activity/project requires an instrument, such as LabPro, to measure and record a heart rate.)

One measure of cardiovascular fitness is how quickly the heart rate (HR) drops during the first three minutes after exercise and also how long it takes the heart rate to return to “normal,” what is called the sedentary heart rate. The time from the end of the exercise until the normal heart rate is obtained is called the *recovery time*.

Contact your physical education department for their standard or guideline data on: Exercise HR, 3 minute HR, Sedentary HR, and Recovery Time. Compare the HR of one or more students against this standard by doing the following:

- a. Measure the student’s normal heart rate.
- b. Have the student conduct a vigorous exercise for five minutes and then collect heart rate data for the next three minutes.
- c. Have groups of students model the heart rate data.
- d. Use the heart rate model to predict the recovery time.
- e. Compare the results to the standard or guideline results.

Complete the activity/project by having each group critique the experiment and their model. How reliable is the result? What are the strengths and weaknesses of the model? What assumptions were made in developing the model? etc.

An alternative to collecting data is to use the following data from a similar activity that was carried out in a class at West Point this fall. The question posed to the class was: Are incoming cadets (Plebes) in better cardiovascular shape than senior cadets?

The Physical Education department provided the following data averaged over the senior class.

Exercise HR	184 bpm
3 Minute HR	124 bpm
Sedentary HR	68 bpm
Recovery Time	9 minutes

A Plebe was recruited and his heart rate data was obtained during class. After measuring his sedentary heart rate (75 bpm), he was sent to run up and down a flight of stairs for five minutes. When he returned to class, the following data was obtained using LabPro and a computer.

Time (min)	Heart Rate (bpm)
0.134	181.8319472
0.268	173.6367575
0.402	172.2158703
0.536	163.8150485
0.67	160.4477429
0.804	153.8698733
0.938	150.0037194
1.072	146.3272227
1.206	142.5035335
1.34	139.6956319
1.474	135.6851702
1.608	132.3432303
1.742	130.7893923
1.876	128.1471051
2.01	125.609584
2.144	124.3781393

2.278	122.4645295
2.412	118.9704246
2.546	125.0094219
2.68	122.4645295
2.814	118.8089375
2.948	117.6384269
3.082	117.061887
3.216	116.0002203

[7] Notices

1. A panel session on *Refocusing College Algebra* will be held Monday morning at 9:00 am on January 8, 2007 as part of the Joint Mathematics Meeting in New Orleans. The panelists will be representatives of the six HBCU schools participating in the NSF funded HBCU Retreat and Follow On program. The panelists will discuss their experiences in refocusing their college algebra courses.
2. Laurette Foster and Don Small will present a minicourse at the Joint Mathematics Meetings in New Orleans, January 5-8, 2007. The title of the minicourse is *Contemporary College Algebra: A Refocused College Algebra Course*. Part A will be offered on Friday, 2:15 to 4:15 pm and Part B will be offered on Sunday, 3:30 to 5:30 pm.
3. Deadline for contributions to the November Newsletter is Wednesday, November 1, 2006. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.
4. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.