

Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
Number 67, February 2006

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NOTICE!!!

Starting with the April issue, the *Vision-Potential* Newsletter will be distributed electronically. In order to continue receiving the Newsletter, send your e-mail address to Don Small, don-small@usma.edu.

[1] HBCU Retreat and Follow-on Program

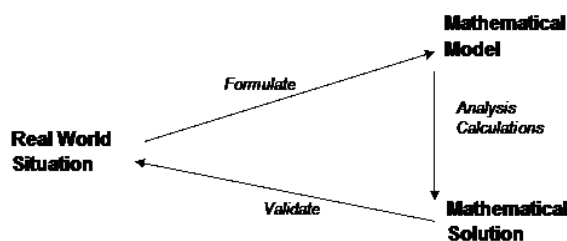
The National Science Foundation (NSF) and the Army Research Office (ARO) have funded a new program to assist eleven HBCUs to refocus their college algebra or calculus programs. The program will be administered by the U.S. Military Academy.

There have been widespread efforts over the past fifteen years to rejuvenate and refocus college algebra and calculus programs. Pedagogically the reforms have focused on changing instruction from being teacher centered

* Supported by the U.S. Military Academy.

to being student centered with emphasis on small group work. Course contents have been revised to include elementary data analysis, multiple representations, and modeling real world situations. The new curriculums are responsive to student needs with respect to developing communication skills, quantitative literacy, and critical thinking. Problem solving in the modeling sense rather than the exercise sense is emphasized. Appropriate use of technology for teaching and learning mathematics is an integral part of the revised curriculums. In general, these curriculums are directed toward leveraging the power of human reasoning to formulate and validate while using the power of technology to calculate.

Problem Solving/Modeling Process



The importance of the new curriculums was articulated in the 2004 MAA report, *Voices of the Partner Disciplines*. Faculty members in other disciplines recommended that mathematics departments “Replace traditional college algebra courses with courses stressing problem solving, mathematical modeling, descriptive statistics, and applications in the appropriate technical areas” and to

“de-emphasize intricate algebraic manipulations.”

Program Goals

The goals of this three-year program are to: (1) Provide a structured opportunity for eleven HBCUs to reform their college algebra or calculus curriculum and (2) Develop and pilot test reform programs. Six HBCUs will be selected this spring (2006) to participate in a curriculum retreat at the U.S. Military Academy (USMA). During the retreat, each school team, assisted by a mentor, will finalize a reform plan suitable to their school and develop an implementation strategy. The “Follow-On” portion of the program will consist of the mentors making two site visits to each school and presentations at the Joint Mathematics Meetings.

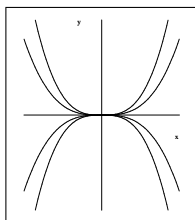
The program will cover the travel expenses as well as provide a small stipend to those attending the Retreat. In addition each participating school will be offered an opportunity to apply for a \$5,000 mini-grant to facilitate implementation of their reform program.

To obtain an application form or to ask for further details, contact Don Small (phone: 845-938-2227; e-mail: don-small@usma.edu or Dennis Davenport (phone: 513.529.3555; e-mail: davenpde@muohio.edu).

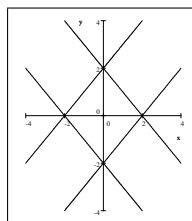
[2] Class Activity: Graph Transformations

Cameron Cooper
Fort Lewis College

a. Determine four transformations (e.g., shift, scale, reflect) of $f(x) = x^3$ whose multi-graph is:



b. Determine four transformations (e.g., shift, scale, reflect) of $f(x) = |x|$ whose multi-graph is:



[3] Project: Slopes of Perpendicular Lines

An over simplification of the process of developing mathematical results can be described as a two stage process. The first stage is discovery in which a person experiments, collects data, and makes conjectures. The second stage is theoretical in which mathematical facts and techniques are used to verify or dispute the conjectures. This project illustrates these two stages.

Discovery Stage.

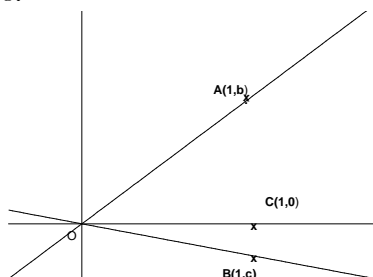
- Let line A pass through the origin and form an angle of 30° with the positive horizontal axis and let line B pass through the origin and form a 120° with the positive horizontal axis. Compare the slopes of lines A and B .
- Repeat Part a with the angle for line A changed to 45° and the angle for B changed to 135° .
- Repeat Part a with the angle for line A changed to 60° and the angle for B changed to 150° .
- Repeat Part a with the angle for line A changed to 45° and the angle for B changed to 120° .

Based on your results in Parts a-d, make a conjecture concerning the relationship between slopes of perpendicular lines. (If a conjecture is not evident, make up and do some more experimental exercises.)

Theoretical Stage.

Outline of Proof: Slopes of Perpendicular Lines are Negative Reciprocals of each other.

Consider two lines $L_1 : y = m_1x$ and $L_2 : y = m_2x$ that are perpendicular to each other. Show that the slopes of the two lines are negative reciprocals, that is, $m_1m_2 = -1$. Refer to the following figure, assume the angle, $\angle AOB$, formed by the two lines is a right angle.



- Explain why $m_1 = b$ and $m_2 = c$.
- Apply the Pythagorean theorem to triangles OAB , OAC , and OCB .
- Simplify the equation: $(\text{distance } AC) + (\text{distance } CB) = (\text{distance } AB)$ to show that $m_1m_2 = -1$.

Discuss why this proof covers the cases where the perpendicular lines meet at some point other than the origin.

Explain why this argument does not hold when the lines are not perpendicular to each other.

[4] **Project:** *Handshake Problem*

Diana Perdue
Virginia State University

This activity is useful for modeling n th term patterns that are quadratic – I suggest doing a prior activity where the pattern is linear. In this small group activity, students form “parties” of different numbers to explore variations of the classic Handshake Problem: “You attend a party with 49 other people. Everyone shakes hands with everyone else.

How many total handshakes happened at the party?”

Guide students in utilizing several of Polya’s problem-solving heuristics, specifically:

- Use smaller numbers
- Act it out
- Make a table
- Look for a pattern

Students are given a handout that guides them in this process. The handout has a table similar to the one shown below:

# People in Party	# of Handshakes
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

They are then directed to “act out” the parties by getting in groups of the given number needed in the party, shaking hands with everyone, and finding the total number of handshakes for each party. Emphasize that correct data collection is key to this process to ensure that all people in the party carefully tally the total number of handshakes. Students will probably discover that they can predict the number of handshakes in the next party given the people and handshakes in the previous party. The students know that their “goal” is to be able to solve the given problem (e.g. party of 50 people) and they quickly discover the disadvantages of an indirect pattern (to find n , they must know $n-1$; to find $n-1$, they must know $n-2$, etc.) and are highly motivated to find a direct rule for finding the n th term. The process to find this rule can be trial and error, graphing points and fitting a curve, or a combination of these. Follow-on

problems can include connections to Geometry like Pick's Formula for area of polygons using Geoboards and the formula for the number of diagonals in an n-sided polygon.

[5] College Algebra Workshops

PREP Workshop

(PProfessional Enhancement Program)

Sponsored by the Mathematical Association of America and Florida Gulf Coast University.

Location: Florida Gulf Coast University, Ft. Meyers, FL

May 22-25, 2006. Participants pay their own travel, room (dorm), and board at Florida Gulf Coast University.

Facilitators: Don Small (U.S. Military Academy), Norma Agras (Miami Dade College), Yvette Stepanian (Virginia Commonwealth University).

Program: Refocus College Algebra. Pedagogically, the program will focus on changing instruction from being teacher centered to being student centered with emphasis on small group work. Course content will emphasize elementary data analysis, multiple representations, and modeling real world situations. Problem solving in the modeling sense rather than the exercise sense will be emphasized. Refocused programs are responsive to student needs with respect to developing communication skills, quantitative literacy, and critical thinking.

Contact: Diane Schmidt, dschmidt@fgcu.edu

Connecticut Community College Workshop

Sponsored by Gateway Community College

Location: Gateway Community College, North Haven, CT

May 31 - June 2, 2006. Participants pay their own expenses.

Facilitator: Don Small (U.S. Military Academy)

Program: Similar to the program of the PREP workshop previously described.

Contact: Miguel Garcia,
MGarcia@gwcc.comnet.edu
(203) 285-2358 or (203) 494-9987

HBCU Retreat

Sponsored by NSF, ARO, USMA

Location: U.S. Military Academy, West Point, NY
June 5-8, 2006

Mentors: Don Small, Donald Outing, Archie Wilmer, Tony Johnson (all of U.S. Military Academy); Laurette Foster (Prairie View A&M); Dennis Davenport (Miami University of Ohio).

Program: See the lead article in this Newsletter.

[6] Notices

1. Deadline for contributions to the March Newsletter is March 1, 2006. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed. Please e-mail contributions to don-small@usma.edu.
2. To subscribe to this Newsletter, write to Don Small, Department of Mathematical Sciences, U.S. Military Academy, West Point, NY 10996 or e-mail at don-small@usma.edu.