

Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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NOTICE!!!

Starting with the April issue, the *Vision-Potential* Newsletter will be distributed electronically. In order to continue receiving the Newsletter, send your e-mail address to Don Small, <don-small@usma.edu>.

This issue is devoted to providing examples of final exam questions. Instructors are encouraged to modify the questions according to their classes and to use the questions to generate more questions for their students. Two of the goals of the Contemporary College Algebra Program are to develop students' communication skills and to empower students to use mathematics to quantify real-life problems. Both goals require students to develop the facility to translate from sentence form to symbolic form and conversely. Thus we intentionally state the majority of the problems in sentence form.

* Supported by the National Science Foundation and the U.S. Military Academy.

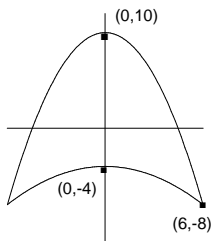
[1] Test Questions - Algebraic Skill

1. (Yvette Stepanian, Virginia Commonwealth Univ.) The price of a gift plus 8% delivery charge comes to a total of \$18.90. What was the price of the gift?
2. The formula for summing the geometric series $1 + r + r^2 + \dots + r^{n-1}$ is:
$$1 + r + r^2 + \dots + r^{n-1} = \frac{1-r^n}{1-r}$$
 for $r \neq 1$.
 - (a) Write down the formula for the sum of the geometric series:
$$3 + 3^2 + 3^3 + \dots + 3^n$$
 - (b) Verify your formula in Part a for the series $3 + 3^2 + 3^3$.
3. Compute the distance between the points (1,2) and (5,1).
4. Determine the equation of a line that separates the plane into two half-planes such that the point (3,5) lies in one half-plane and point (2,4) lies in the other half-plane.
5. A salesperson is offered two salary plans. The first provides a \$1000 per month base salary along with a 5% commission on sales for that month. The second offers a base salary of \$1500 per month along with a 4% commission on sales for that month. Determine the amount of sales that provide the same income under the two plans.

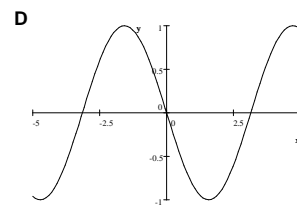
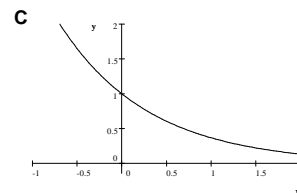
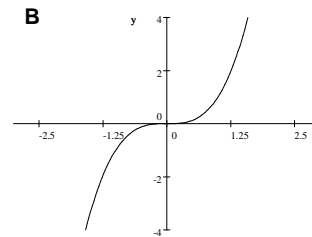
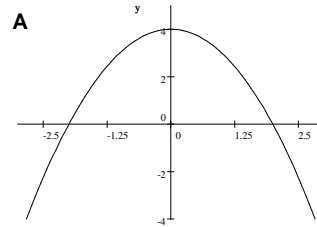
- Two lines are perpendicular provided their slopes are negative reciprocals of each other (e.g., $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals of each other). Determine the equation of the line that contains the point $(2, 3)$ and is perpendicular to the line $4y + 5x = 12$.
- Suppose your course grade depends on three, one hour tests and a two hour final exam. The hour tests are all weighted equally and the weight of the final exam is twice that of an hour test. If your hour test scores have been 77, 61, and 70, what score do you need to receive on the final exam to have a course average of 74?
- A farmer figures that, on average, if he plants x tomato plants, he will harvest $2x^3 + 20x$ pounds of tomatoes. What is the minimum number of tomato plants he needs to be assured of raising 2,000 pounds of tomatoes?

[2] Test Questions - Graphing

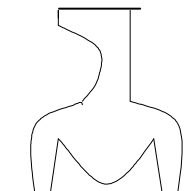
- Determine the slope of the line passing through the origin and making a 45° angle with the positive horizontal axis.
- (Cameron Cooper, Ft. Lewis College) Determine two functions that are transformations (e.g., shift, scale, reflect) of $f(x) = x^2$ whose graphs outline the following arrow head shape.



- Describe the points in the xy -plane satisfying the equation $y \geq 3x - 2$. Illustrate your answer with a sketch.
- For each of the following plots, give a function whose graph approximates the plot.

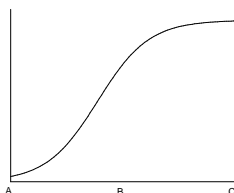


- Water is poured at a constant rate into a modern vase whose cross section is pictured below. Sketch a graph of the depth of the water in the vase as a function of time.



[3] Test Questions - Writing

1. Average and median are measures of central tendency for a numerical data set. Which of the two measures is more sensitive to outliers in the data set? Answer the question and then write a few sentences explaining your reasoning. Illustrate your reasoning with an example.
2. Let the following graph provide a reasonable picture of the change in the cost of a pair of *Nike* sneakers over the past twenty years. Explain some of the factors that would cause the graph to be concave up over the period from A to B and concave down over the period from B to C.



3. Suppose the cost of benefits is a linear function of the number of employees. Explain why the slope of the linear function represents the cost of benefits to a single employee.
4. Consider a company whose daily sales are continually increasing. Describe how the graph of the daily sales compares with the graph of the average number of sales. (E.g., does the graph of the average number of sales lie above, below, or does it intersect the graph of the daily sales.) Explain your reasoning.
5. Describe what information about a function a graph can supply and what information a graph cannot supply.

[4] **Test Questions - True-False** (If the statement is true, explain why it is true. If the statement is false, give an example to show that it is false.)

1. If the distance between the points $X(a, b)$ and $Y(c, d)$ is D , then the distance between $Z(ka, kb)$ and $Y(c, d)$ is $|k|D$.
2. If the slope of a line passing through the origin making a 60° angle with the positive horizontal axis is $\sqrt{3}$, then the slope of the line passing through the origin making a 120° angle with the positive horizontal axis is $2\sqrt{3}$.
3. The graph of an exponential function has a horizontal asymptote.
4. Every function from set A to set B is a relation, but not every relation from set A to set B is a function.

[5] Test Questions - Modeling

1. **Setting a Sales Target.** Jason, an insurance salesman, receives a base salary of \$2,000 per month plus a 15% commission on his monthly sales over \$10,000. If Jason's goal is to realize a salary of \$5,000 per month, what minimum monthly sales must he obtain?
2. **Potential Savings From a New Furnace.** The increasing cost of fuel has led some people to consider replacing their old furnace with a more efficient one. Furnaces are rated according to their annual fuel utilization efficiency (A.F.U.E.). *The New York Times* (10/16/05) published the following table on potential savings from a

new furnace for every \$100 currently spent.

A.F.U.E, Existing Furnace	Savings/\$100 New Furnace
50%	\$47.36
60%	\$37.80
75%	\$21.10
80%	\$15.80
85%	\$10.50
90%	\$5.30

- (a) Develop a symbolic model for the potential savings per \$100 of current expense from a new furnace.
 - (b) Last year, Don spent \$1800 for fuel for his gas furnace which has an A.F.U.E. rating of 65%. If Don were to spend \$2000 to replace his furnace with one having an A.F.U.E. rating of 90%, how many years would it take for Don to recoup his investment? (Assume that fuel prices remain constant for the next several years.)
 - (c) Determine Don's return on investment using the facts in Part b. (In order to compute the percentage Return on Investment, divide the annual savings by the cost of installation and multiply by 100.)
3. Suppose you expect to receive an annual salary increase of 3%. Develop a model for your projected salary and then use it to determine how many years it will take for your present salary to double.
 4. Suppose you owed \$1000 on your credit card at the beginning of this month and that you will not charge any additional items to your credit card. If the credit card company charges 11% compounded monthly on outstanding balances and you pay the company \$100 at the end of this month, what will be your debt at the beginning of next month?

5. (Yvette Stepanian, Virginia Commonwealth Univ.) For a certain Bob Dylan concert, two types of tickets were available: \$30 tickets and \$20 tickets. If a total of 13,000 tickets were sold and the gross receipts for the concert totaled \$310,000, then how many of each kind were sold?

[6] Notices

1. Past issues of the *Vision - Potential* Newsletter are available on our website: www/ContemporaryCollegeAlgebra.org.
2. Faculty Development Workshop for Contemporary College Algebra: November 3-4, 2005, Prairie View A&M University, Houston, TX. Contact Laurette Foster (lbfooster@pvamu.edu).
3. Faculty Development Workshop for Contemporary College Algebra: December 15-16, 2005, Gateway Community College, North Haven, CT. Contact Miguel Garcia <MGarcia@gwcc.commnet.edu>, (203) 285-2358.
4. To subscribe to this Newsletter, send your name, address, and e-mail address to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or e-mail him at don-small@usma.edu.
5. Deadline for contributions to the January Newsletter is January 1, 2006. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.