Vision - Potential

Vision Within Every Instructor - Potential Within Every Student
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[1] A Comparison Study of the Contemporary College Algebra with Traditional College Algebra

The results cited in this article are taken from Amie Ellington's comparison study of the Contemporary College Algebra (CCA) and traditional college algebra. The full study appears in the September issue of *Primus* under the title "A Modeling-Based College Algebra Course and It's Effect on Student Achievement." The study was based on eight CCA sections (pilot) and eleven traditional sections (control) taught at Virginia Commonwealth University (VCU) in the 2004 Fall semester. Each section had 35 students. The students were not preassigned and did not know the difference between the sections when they enrolled.

The course grade results were:

* Supported by the U.S. Military Academy.

Grade	CCA	Traditional
A	16.90%	12.75%
B	30.99%	17.11%
C	23.94%	19.84%
D	12.32%	12.75%
F	10.22%	17.21%
W	5.63%	20.34%

Table 1

The reduction in the withdrawal rate from 20.34% in the traditional sections to 5.63% in the CCA sections is particularly striking. Furthermore 89.6% of the students in the CCA sections took the final exam (5.63% withdrew and 4.77% of the remaining did not take the final exam). In comparison only 71.33% of the students in the traditional sections took the final exam (20.34% withdrew and 8.33% of the remaining did not take the final exam). Thus not only did the students in the CCA sections earn higher grades, considerably more of them stayed in the course through the final exam than was the case in the traditional sections.

In order to address the question: How well do students completing the CCA course do on skill type questions?, the final exam contained a common portion that all students, CCA and traditional, took. The common portion consisted of ten skill questions and three application questions. All the instructors involved agreed on the questions and this portion of the exam was graded by one person who was not involved with any of the nineteen sections. The grade results were:

	CCA	Traditional
	A,B,C	A,B,C
All Questions	57.14%	41.05%
Skill Questions	53.78%	35.09%
Application Questions	61.29%	47.02%

Table 2

These results clearly demonstrate that students develop the appropriate algebraic skills in courses that de-emphasize skill work in favor of modeling.

The study included an analysis of how well the students did on their follow-on mathematics course during the the Spring semester 2005. VCU students passing college algebra have two options for their follow-on course—precalculus or business calculus. The precalculus is a very traditional, skill oriented course while the business calculus is applications oriented. Table 3 shows that students in the CCA sections did better than those in the traditional sections in the business calculus, but not as well in the precalculus course. The percentages of students passing (A,B,C) were:

Courses	CCA	Traditional
Business Calculus	72.60%	69.57%
Precalculus	56.38%	70.42%

Table 3

A slightly higher percentage of students completing the CCA course (69%) took a follow-on mathematics compared to those in the traditional sections (63%).

Table 4 gives the percentage comparison measured over two semesters, the students in the CCA sections (37.3%) realized a significantly greater success rate then those in the traditional sections (28.3%). This was a result of the higher passing rate and greater percentage of students taking a follow-on mathematics course. The results were:

Follow-on Course	CCA	Traditional
Business Calculus or Precalculus	37.3%	28.3%
Business Calculus	18.7%	8.1%
Precalculus	18.7%	20.2%

Table 4

(The complete article is posted under *Instructor Resources* on our webpage: www.ContemporaryCollege Algebra.org.)

[2] Fun Project: Collisions of Air Hockey Pucks

(Frank Wattenberg, U.S. Military Academy, suggested this project.) The project involves using systems of equations to model the effects when two air hockey pucks collide head-on, taking into account their velocities and weights. The particular questions to be answered are:

1. Suppose a moving puck hits a stationary puck of the same weight head-on. What will happen?

2. Suppose a moving puck hits a stationary puck of heavier (lighter) weight head-on. What will happen?

3. Suppose two pucks of the same weight and traveling at the same velocity collide head-on. What will happen?

4. Suppose two pucks of the same weight and traveling at different velocities collide head-on. What will happen?

It is recommended that this project be introduced and the questions discussed in class before students work in their groups. The discussion should be based on the students intuition and experience. A physical demonstration using different colored tennis balls for pucks would help generate enthusiasm and student interest.

This project provides a nice opportunity to invite a physics instructor to speak to your students about the importance of conservation laws in science. (In Section 2.8, the Law of Conservation of Mass was used to balance chemical equations.) In particular, the introduction to the project needs to include the conservation laws of energy and momentum.

> *Law of Conservation of Energy*: the total kinetic energy of the two pucks is the same before and after the collision. (The kinetic energy of an object with weight w and velocity v is $\frac{wv^2}{2}$.)
>
> *Law of Conservation of Momentum*: the total momentum of the two pucks is the same before and after the collision. (The momentum of an object with weight w and velocity v is wv.)

The students should be encouraged to define variables representing the weight and velocity of each puck before collision and the weight and velocity of each puck after collision. Constructing an equation for each of the two conservation laws gives a system of two equations to solve. (For instance, let w denote weight, u denote the velocity of one puck and v the velocity of the other other puck. Also let subscript one denote time immediately before collision and subscript two denote time immediately after collision. Then for Question 1 the Law of Conservation of Momentum states:

$$wu_1 + w0 = wv_1 + wv_2$$

or

$$u_1 = v_1 + v_2$$

An equation involving $u_1, v_{1,}$ and v_2 can similarly be obtained from the Law of Conservation of Energy. The solution of the resulting system of equations leads to two outcomes. It is important that the students interpret each of the outcomes to see if they are reasonable. (Recall the interpretation leg in the Problem Solving/Modeling Process diagram.)

[3] Activity: Identifying Graphs

For this activity, divide the students into pairs. The instructor sketches a graph on the board and then each pair of students do the following:

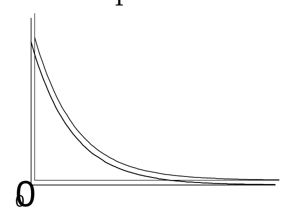
a. Identify the basic shape of the graph.

b. Develop a function representing the function
represented by the graph.

c. Create a “story” corresponding to the graph and their function.

After an appropriate amount of time, the instructor calls on different pairs to share what they have done and to explain their reasoning. Here are some examples:

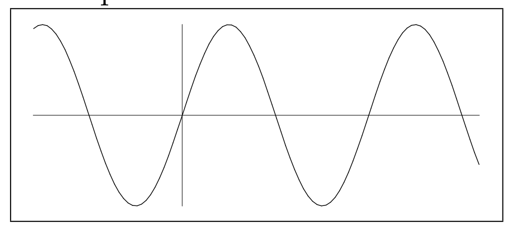
1a. Graph:



1b. Curve is decreasing and concave upward. There is an intercept on the vertical axis, but possibly not on the horizontal axis. The horizontal axis is an asymptote. An approximate function is $f(x) = 5e^{-x}$. (The intercept on the vertical axis rules out a function of the form $g(x) = 1/x$. Why?)

1c. Interpretation. Water draining out of a bathtub—vertical axis represents the depth of the water in inches over the drain and the horizontal axis represents time in minutes.

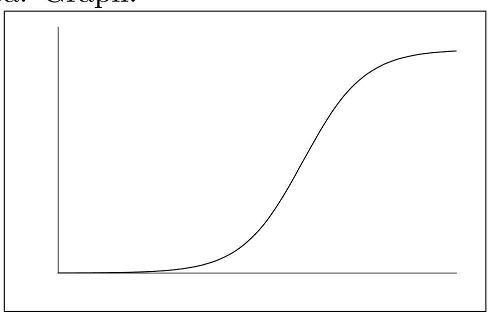
2a. Graph:



2b. The curve suggests a periodic function, say sine or cosine. Since the curve passes through the origin, $f(x) = \sin(x)$ would be a reasonable approximation.

2c. Interpretation. The number of hours of possible sunlight in Buffalo, New York. Hours are measured on the vertical axis and days of the year on the horizontal axis

3a. Graph:



3b. Curve is increasing, concave upward at first and then changes to concave downward. There appears to be two horizontal asymptotes, the horizontal axis and where the cure appears to flatten out. A logistic curve would be a reasonable approximation, $f(x) = \frac{20}{1+20e^{-(x-5)}}$. (A reasonable approach would be to approximate with a two-part function. The first part could be a quadratic function or an exponential function and the second part could be a negative quadratic shifted up and to the right or the negative of a negative exponential shifted up and to the right.)

3c. Interpretation. The amount of fish mass in an aquarium.

[4] In-class Activity — Lines & Half-planes

Ask the class to explain what is meant by saying "A straight line divides the plane into two half planes." When this is understood, divide the class into pairs.

1. Ask each pair to determine the equation of a line that does not include the point (2,3). (Different pairs will probably have different lines.) Then ask each pair to use an inequality to analytically describe the half-plane that contains the point (2,3). Ask one of the pairs to explain to the class what they did and their reasoning.

2. Ask each pair to determine a second line that does not contain the point (2, 3) and is not parallel to their first line. Using inequalities, analytically describe the "half-plane" that contains the point (2,3). Ask one of the pairs to explain to the class what they did and their reasoning.

[5] Notices

1. Past issues of the *Vision - Potential* Newsletter are available on our website: www//ContemporaryCollegeAlgebra.org.

2. Faculty Development Workshop for Contemporary College Algebra: Nov. 3-4, 2005, Houston, TX. Contact Laurette Foster (lbfoster@pvamu.edu).

3. **REQUEST**: The November issue of our Newsletter will focus on final exam questions. Please send in samples of your final exams questions so that they can be shared through the Newsletter.

4. Deadline for contributions to the November Newsletter is November 1, 2005. In addition to final exam questions, opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

5. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.