Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

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[1] Problem Solving Heuristics

There is wide spread agreement on the importance of students developing effective problem solving skills. However, there is considerably less agreement on what these skills are and even less agreement on how to teach problem solving. The purpose of this article is to introduce or reintroduce readers to a few of George Polya's heuristics on problem solving and to encourage readers to pursue the subject further by reading his two volume set entitled *Mathematical Discovery, On Understanding, Learning, and Teaching Problem Solving,* (1962) published by John Wiley and Sons, New York.

* Supported by the U.S. Military Academy.

Polya writes in the Preface to volume I "Solving problems is a practical art, like swimming or skiing, or playing the piano: you can learn it only by imitation and practice. If you wish to learn swimming you have to go into the water, and if you wish to become a problem solver you have to solve problems."

Reading a problem to clearly identify the three basic parts is an important, although often overlooked heuristic. These parts are:

Data—the information given in the problem,
Unknown Variables—representing what the problem is asking for, and
Conditions or *Constraints*—that link the data to the unknowns.

Example. The label on a half-pint carton of 2% milk claims 38% less fat than whole milk. If a half-pint serving of 2% milk contains 5 grams of fat, how many grams of fat are there in a half-pint serving of whole milk?

> Data: half-pint of 2% milk contains 5 grams of fat.
>
> Unknown Variable: x = the number of grams of fat in a half-pint serving of whole milk.
>
> Condition: 2% milk has 38% less fat than whole milk.

Model: Translate the condition into an equation by expressing the central idea in two different ways and then setting the two ways equal to each other. In this example, express the amount of fat in a half-pint of 2% milk in two different ways— 5 grams and $(1-0.38)x$. This yields the equation $(1-0.38)x = 5$.

A useful insight to forming an equation model is given by noting that the two sides of an equation represent the same thing. Thus to form an equation, find two different ways of representing the same quantity and then set them equal to each other.

Successive approximations is another useful heuristic. Although sometimes called "guess and check" by students, forming a successive approximation involves much more than guessing. The process involves making an initial attempt, noting the resulting errors, modifying the attempt to reduce the errors, and then trying again. This procedure is repeated until a satisfactory solution is obtained. Graphically fitting a curve to a scatter plot (see [6]) is a good example of the use of successive approximations.

Polya'a *Wishful Thinking* heuristic is to assume the problem has been solved and work backwards. The nature or form of a solution often provides insights into what is needed to solve the problem.

Example. Determine the intersection of the two planes, $x + y + z = 2$ and $x - z = 3$. The assumption that the problem has been solved leads to the realization that the intersection of the two planes is a straight line in three-dimensional space. This realization, in turn, leads to two others. One is that the answer is given by a set of parametric equations. The other is that since two points are sufficient to determine the equation of the line passing through them, the solution could be obtained by finding two points whose coordinates satisfy the equations of the two planes. These two ideas suggest solving the equations of the two planes as a system of two equations in three unknowns. The solution of two of the unknowns in terms of the third unknown yields the desired set of parametric equations.(The third unknown serves as the parameter.)

For more, read Polya.

[2] Read the Text

The following is a quotation from Isaac Newton describing how he learned mathematics. The quote and its message about how to learn is worth sharing with students.

> "Took Descartes's Geometry in hand, thro he had been told it would be very difficult, read some ten pages in it, then stopt, began again, went a little further than the first time, stopt again, went back again to the beginning, read on till by degrees he made himself master of the whole, to that degree that he understood Descartes's Geometry better than he had done Euclid."
>
> Source: *The Mathematical Papers of Isaac Newton*, vol 1, p.5-6

One should remember that Newton was in the smart group. Everyone should expect to have to read their text more than once, it is a proven way of learning.

[3] Optimization Problems

The technology of the graphing calculator or a computer algebra system has moved optimization problems from the realm of calculus to the realm of college algebra. The following four optimization problems provide challenging problem solving scenarios of increasing difficulty for college algebra students.

Most shops involved in cutting smaller pieces of material from larger pieces, such as carpentry or upholstery or metal working, have a scrap pile, a collection of leftover pieces. Because the materials in these scrap piles have already been "paid for," money can usually be saved by using a piece from the scrap pile rather than cutting into new stock. Since many times the craftsman starts with a rectangular shape piece of material, the value of a scrap piece could be determined by the largest rectangle contained in the piece. However, determining the size of the largest rectangle in a piece of scrap can be a challenging problem as illustrated by the following problems.

1. Consider a region bounded by the graphs of $f(x) = e^x$, x=2, y=0, and x=0. Determine the dimensions and area of the largest rectangle contained in this region.

2. Consider a region bounded by the graphs of $f(x) = \cos(x)$ and the interval $[-\pi/2, \pi/2]$ on the x-axis. Determine the dimensions and area of the largest rectangle contained in this region.

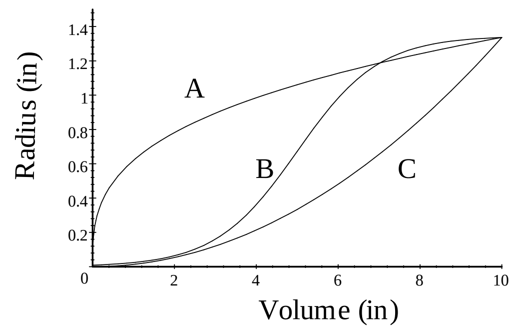
3. Consider a region bounded by the graphs of $f(x) = \sin(x)$ and the interval $[0, \pi]$ on the x-axis. Determine the dimensions and area of the largest rectangle contained in this region.

4. Consider a region bounded by the graphs of $f(x) = x^2$, $g(x) = 20\ e^{-x}$, and the x-axis. Determine the dimensions and area of the largest rectangle contained in this region.

[4] Queries

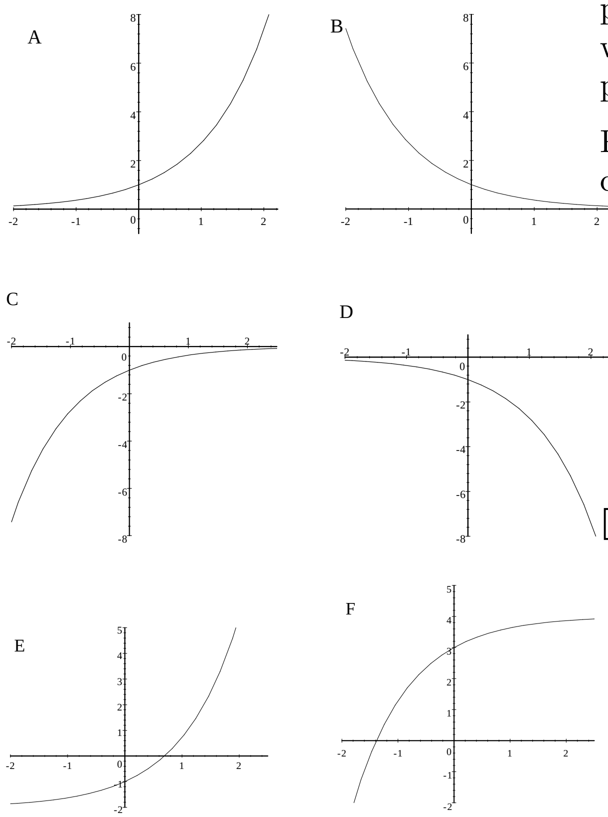
1. (Submitted by Shane Brewer of Eastern Utah State College) Highway engineers use the term *grade* to describe the slope or steepness of a road. Highway signs alerting motorists of a steep hill ahead usually express the grade in terms of percentages. For example, an 8% grade would indicate that the road rises 8 feet, over a 100 foot horizontal distance. Clearly a 10% grade indicates a steeper hill than does a 6% grade. If you saw a sign "Warning: 100% Grade Ahead," what would you expect?

2. Think about blowing up a (spherical) beach ball. In particular, think about how the radius of the ball increases as the volume increases. Consider the three curves in the following multiplot. Do any of them approximate the shape of the radius as a function of the volume or is the radius curve completely different? Write a few sentences describing your thought process and how it led you to an answer.



[5] Recognizing Plots

In the following set of plots, A is the plot of $f(x) = e^x$. The other plots were obtained by shifting and/or scaling the plot in A. Write the function expression for each of the other plots.



[6] The Wind Doth Blow

The cost of generating electricity from wind has decreased dramatically over the past 20 years primarily due to (a) improved turbine design, (2) taller towers to reach higher wind speeds (potential wind energy is proportional to the cube of wind speed), (3) longer blade length (the swept area is proportional to the square of the blade length). The American Wind Energy Association predicts the proportion of U.S. electricity produced from wind will grow from one percent today to 20 percent by 2020.

Plot the following data and graphically fit a curve to the scatter plot.

Year	#Megawatts	Year	Megawatts
1982	72	1991	1571
1983	214	1993	1643
1985	1000	1995	1714
1986	1214	1998	1857
1987	1357	2000	2786
1990	1500	2002	4714

[7] Notices

1. Prior Newsletters are archived in www.ContemporaryCollegeAlgebra.org

2. Deadline for contributions to the April Newsletter is Monday, April 5, 2004. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

3. To subscribe to this Newsletter, write Don Small, Dept. of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.