Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

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[1] Functions

The concept of relation, in general, and function, in particular, is of fundamental importance in mathematics. Modeling real-world scenarios relies on being able to identify relations between different factors. The price of a concert ticket is related to the location of the seat. The cost of a speeding ticket is related to how fast the offender was traveling. The amount of a credit card charge is related to the balance due and the card's interest rate. When these relations are *dependency* relations, it is important to identify the *cause* (independent variable) and the *effect* (dependent variable) and, if possible, to understand how the cause produces the effect. Knowing the *unique* effect of a cause allows one to make predictions. The uniqueness of the output (effect) for a given input (cause) is essential for making predictions. Thus the importance of the function concept.

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Trouble shooting, where you know the effect and you seek the cause, depends on the uniqueness aspect of a function relation. The oil light illuminating on your dashboard is the result (effect) of insufficient oil in the crankcase. Because the function relation between the level of oil in the crankcase and the illumination of the oil light is a one-to-one relation, the inverse relation is also a function. Thus the reason the oil light illuminated is clear as is the remedy. This is not the case for the engine light illuminating on your dashboard as there are several causes that could illuminate the light. Thus the inverse relation is not a function and therefore the particular reason for the illumination is not necessarily evident.

The rule describing how a function maps (pairs) an element in its domain to an element in its range may be given in words, with a table, with a graph, or with an equation. Functions in real-world scenarios are usually given by one of the first three means. Unfortunately, however, textbooks usually display functions as equations, misleading students to think of a function as just an equation.

[2] Scrabble Point Functions

(This problem was adapted from a sample problem that Allan Rossman prepared for the forthcoming text *Workshop Precalculus* by Nancy Hastings and Allan Rossman.)

The purpose of this class activity is to generate discussions about functions. Each student is given a page containing the following table of Scrabble Points and four columns labeled: Name, # Letters, # Scrabble Points, and Ratio. "Letters" refer to the letters in the last names of the students and "Ratio" refers to the ratio of the number of scrabble points to the number of letters.

S	B	C	D	E	F	G	H	I
1	3	3	2	1	4	2	4	1
J	K	L	M	N	O	P	Q	R
8	5	1	3	1	1	3	10	1
S	T	U	V	W	X	Y	Z	
1	1	1	4	4	8	4	10	

Each student fills in the four columns. If projection facilities exist, it is best to project the four-column table on a screen with the names already listed. A student (or instructor) fills in the table as students provide the pertinent information.

For each of the following, determine if the relation described is a function. If it is, describe the domain and range of the function. Also explain whether or not the function has an inverse. If the relation is not a function, explain why it is not.

1. The relation of Scrabble Point to Name.
2. The relation of Scrabble Point to Letters.
3. The relation of Letters to Name.
4. The relation of Name to Letters.
5. The relation of Ratio to Scrabble Points and Letters.
6. The relation of Scrabble Points and Letters to Names.

[3] Change Function

Define a change function that maps an amount of money from one to one hundred cents into the least number of coins (penny, nickel,dime, quarter) that are needed to make that amount. Describe the function in a two-column table. Let the first column list the number of cents from one to one hundred and let the second column list the corresponding numbers of coins. Discuss your function. (Is your change function increasing (decreasing)? Is your change function periodic? What is the maximum (minimum) of your change function? Does your change function have an inverse function? What does the graph look like?)

A variation on the preceding would be to replace the second column with four columns—#penny, #nickel, #dime, #quarter. Now investigate relations between different columns. For example, a "Quarter Change" function would map fifty cents into two quarters. Compare and contrast the "Penny Change" function, "Nickel Change" function, "Dime Change" function, and "Quarter Change" function by forming a multiplot of their four graphs. Denote each row of the four columns as a four-tuple, (#penny, #nickel, #dime, #quarter). Are these four-tuples a function of the money amount in the first column? Is the money amount in the first column a function of these four-tuples? Explain.

[4] Windchill

"I don't mind the cold temperature, but it is the wind that bothers." Have you ever said this or heard someone else say it? However, it is not just the wind because on a hot summer day a brisk wind is welcome. In colder climates, the weather forecasts usually include a windchill reading. Windchill combines temperature and wind speed as illustrated in the following table. (This table is a portion of the full table that defines windchill.) The column headings are temperatures (^{o}F) and the row headings are wind speeds (mph).

Windchill Table

20	15	10	5	0	-5	-10	-15	-20
13	7	1	-5	-11	-16	-22	-28	-34
9	3	-4	-10	-16	-22	-28	-35	-41
6	0	07	-13	-19	-26	-32	-39	-45
4	-2	-9	-15	-22	-29	-35	-37	-44
3	-4	-11	-17	-24	-31	-37	-44	-51
1	-5	-12	-19	-26	-33	-39	-46	-53
0	-7	-14	-21	-27	-34	-41	-48	-55
-1	-8	-15	-22	-28	-36	-43	-50	-57
-2	-9	-16	-23	-30	-37	-44	-51	-58

Does this table define a windchill function? If it does, discuss the function. For example, describe its domain and range, give its maximum and minimum values, describe its graph, describe the rule telling how to map an element in the domain to a unique element in the range, etc. Illustrate the mapping of the function by selecting an element in the domain and giving its corresponding element in the range.

[5] Interpreting Plots

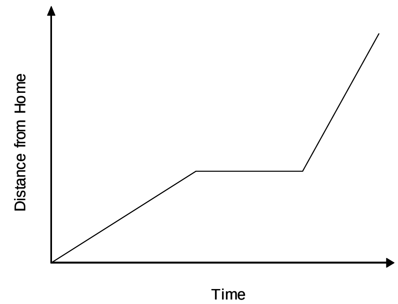
Match each of the scenarios with one of the graphs.

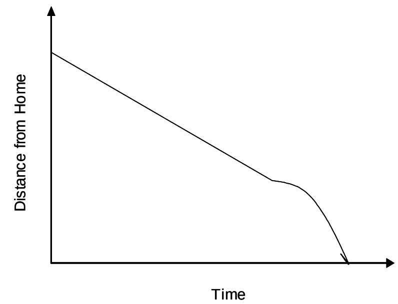
a. Walking to the store I met a friend who was on her way home and we chatted for awhile.

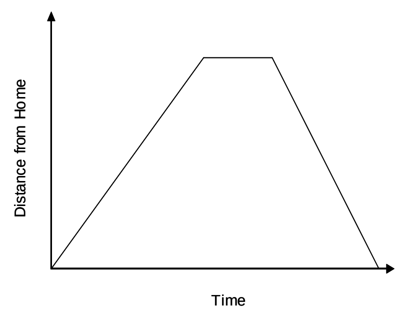
b. On the way home, I heard thunder and hurried to get home before it rained.

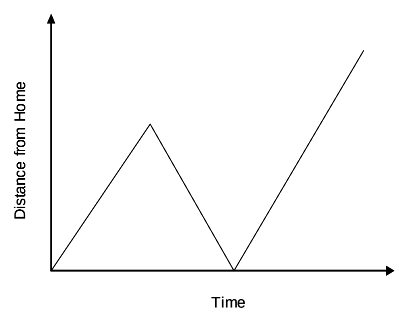
c. On my trip to the store, I walked twice as fast coming home as I did going.

d. On my way to the store, I realized that I had left my purse at home and had to return home to get it before going to the store.









[6] Magic Square

Submitted by
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Determining the entries in a magic square is a nice application of a system of linear equations. Consider a 3 by 3 magic square. It is a square array of the integers 1, 2, 3, . . . , 9 in which the three rows, three columns, and two diagonals all sum to the same value. This represents a system of eight linear equations.

Set up the system of linear equations and then solve for the unknowns $u, v, w, x, y,$and z for the magic square

$$\begin{bmatrix} 4 & \text{u} & \text{v} \\ \text{w} & 5 & \text{x} \\ \text{y} & \text{z} & 6 \end{bmatrix}$$

Note that since the elements on one of the diagonals are given, there will only be seven equations in the system, however there are only six unknowns. What does this imply about the solution(s)? Furthermore since the elements must be unique, none of the six unknowns can have the value 4 or 5 or 6. If the reflection or rotation of a magic square is considered to be the same magic square as the original, how many solutions are there?

Hint. Form the augmented matrix of the system and then reduce it using the $rref$ command on the calculator. Solve for each of the unknowns in terms of z and then apply the constraint that none of the unknowns can have the values 4, 5, or 6.

[7] Challenge Problems

The following problems were presented by Penny Dunham, Muhlenberg College, during the 2004 Joint Mathematics Meetings in Phoenix, AZ.

1. “The Four 4s Challenge: Using exactly four 4s—no more and no less and the operations of addition, subtraction, multiplication, and division as well as parentheses, write expressions for the numbers 1 through 10.” Example: 1 = (4/4) + 4 - 4.

2. “The Nine Digit Challenge: Place the digits 1 through 9 (each exactly once) to make a true statement in the form of a sum XXX + XXX + XXX = XXX. (From Wonderful Ideas, 1995-96 Sampler)”

[8] Notices

1. <u>Deadline</u> for contributions to the March Newsletter is Monday, 2, 2004. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

2. To subscribe to this Newsletter, write Don Small, Dept. of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.