Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Searching for a Pattern

Pattern recognition is often presented as one of the major characteristics of mathematics, in fact Professor Lynn Steen of St. Olaf's College wrote

> "As biology is a science of living organisms and physics is a science of matter and energy, so *mathematics is a science of patterns.*" ("The Science of Patterns," Science, 240(April 29,1988), 611-616)

There is certainly no recipe that always works for discovering a pattern or even establishing that a pattern exists. However, there are some approaches that have often been productive in discovering patterns. This article illustrates one such approach. It is an iterative approach that works often enough to be worth our attention.

Problem. Determine the sum of the following:

$\frac{1}{1*2}+\frac{1}{2*3}+\frac{1}{3*4}+\frac{1}{4*5}+...+\frac{1}{n*(n+1)}$ for any positive integer n.

For a small value of n, we could just add up the terms. However if $n = 100$ or if $n = 1,000$, adding the terms would be a tedious exercise that would not provide very much insight and would certainly not answer the question for an arbitrary n. Thus we try to discover a pattern that would help lead us to a solution.

Solution Approach. We begin by defining variables, let:

$S(n)=\frac{1}{1*2}+\frac{1}{2*3}+\frac{1}{3*4}+\frac{1}{4*5}+...+\frac{1}{n*(n+1)}$

n represents a positive integer.

Now iterate $S(n)$ for several values of n beginning with $n = 1$.

$S(1)=\frac{1}{1*2}=\frac{1}{2}$
$S(2)=\frac{1}{1*2}+\frac{1}{2*3}=\frac{1}{2}+\frac{1}{2*3}=\frac{2}{3}$
$S(3)=\frac{1}{1*2}+\frac{1}{2*3}+\frac{1}{3*4}=\frac{2}{3}+\frac{1}{3*4}=\frac{3}{4}$
$S(4)=\frac{1}{1*2}+\frac{1}{2*3}+\frac{1}{3*4}+\frac{1}{4*5}=\frac{3}{4}+\frac{1}{4*5}=\frac{4}{5}$

Continue the iteration until a conjecture can be made. Forming a conjecture often involves associating parts of the outcome with the input. In this case, the value of $S(n)$ is the output and n is the input. The iterations suggest the pattern $S(n)=\frac{n}{n+1}$.

Although a proof of our conjecture may be beyond our reach at present (it involves Mathematical Induction), we can gain confidence in the truth of our conjecture by "checking it out" for several different values of n.

* Supported by the U.S. Military Academy, West Point, New York.

[2] Daylight in Eastern Montana

Is the theoretical number of daylight hours in Eastern Montana more than, equal to, or less than the theoretical number of non-daylight hours? The following table gives the 2003 official times of sunrise and sunset for Eastern Montana as adopted by the Montana Fish, and Park Commission.

Date	Sunrise	Sunset
Jan. 31	7:26	5:07
Feb. 29	6:43	5:49
Mar. 31	5:45	6:31
Apr. 30	5:51	8:11
May 31	5:15	8:47
June 30	5:14	9:00
July 31	5:43	8:36
Aug. 31	6:22	7:45
Sep. 30	7:00	6:46
Oct. 31	6:43	4:51
Nov. 30	7:24	4:20
Dec. 31	7:45	4:27

Write a few sentences to describe your reasoning process in determining your answer. Does your answer depend on an exact or an estimate for the average number of hours of daylight? If your average number of hours is an estimate, explain whether it is too high or too low. Explain how you accounted for the fact that 7 months have 31 days, 4 have 30 days, and 1 has 29 days in determining that your estimate was too high or too low.

[3] House Lot

(This is a small group project that illustrates a type of problem that a surveyor in a zoning office may confront.)

In many communities, Zoning Boards have established building lot restrictions to limit over-crowding, to limit urban sprawl, to preserve the "nature of the community," etc. The existence of zoning regulations often present potential home owners, surveyors, and draftsmen with problems similar to the following one presented by Bill Schoppee, a surveyor.

Problem. Jan is interested in having her dream house built on a triangular shaped lot which contains some wetlands. The lot is bounded by a (straight) road on one side, a property line running perpendicular to the road on the other side, and a stream along the diagonal side of the triangle. When placed on a grid, the vertices of the lot have coordinates (0,100), (0, -170), and (300,100). Wetlands intrude into the property along the side of the perpendicular to the road. The boundary of the wetlands inside the triangular property forms an arc of a circle. The points (0,94.5), (80.5 42.5), and (0, -99.5) lie on this arc.

The following zoning regulations must be met before a building permit can be issued.

> The lot must contain at least 40,000 square feet.
>
> Setbacks: 40 feet from the road, 50 feet from any wetland, and 20 feet from a side boundary. An additional 15 feet must be added to the setback when the boundary is a stream.

If the footprint of Jan's dream house is 60 feet by 30 feet, will she be able to receive a building permit?
Clearly explain your reasoning. If Jan can receive a building permit, describe a possible location for her house.

[5] Municipal Solid Waste–Activities/Projects

(This article provides suggestions for class activities and for small group projects. More information can be obtained from the websitehttp: //www.epa.gov/epaoswer/non-hw/muncpl/timeline.htm.)

Municipal Solid Waste (MSW) - more commonly known as trash or garbage - consists of everyday items such as product packaging, grass clippings, furniture, clothing, bottles, food scraps, newspapers, appliances, paint, and batteries. Management of waste has become a major problem for small towns to large cities to the national government. The questions not only involve the amounts, which are staggering, but also methods of disposal. For example, how to safely dispose of electrical and electronic equipment or spent nuclear fuel rods? The following table shows the increase in MSW over the last forty years measured in millions of tons and the increase in per capita waste measured in pounds per person.

Year	MSW	Per Capita
1960	88.1	2.7
1970	121.1	3.3
1980	151.6	3.7
1990	205.2	4.5
2000	231.9	4.6

Plot this data (two plots), fit a curve to each plot, and then predict the MSW and per capita waste in 2010. Explain how you interpret the small increase in per capita waste during the 1990s.

Use a pie chart to show the distribution of the 231.9 million tons of MSW generated in 2000 given the following data.

Paper	37.4%
Yard Trimmings	12.0%
Food Services	11.2%
Plastics	10.7%
Metals	7.8%
Rubber, Leather, Textiles	6.7%
Glass	5.5%
Wood	5.5%
Other	3.2%

Three major waste management practices have been introduced over the past few years to reduce or treat waste. (1) Source reduction involves design and manufacturing alterations to reduce the amount and toxicity of waste (e.g., two-sided copying, improved transport packing); (2) Recycling that diverts many items such as paper, glass, plastics, and metals from the wastestream; (3) Composting of food waste and yard trimmings to form a humus-like substance. The following table shows the growth of recycling and composting. (The 1950 entries in the following table are estimates to accompany the 1960-1990 data from the Environmental Protection Agency (EPA). Data on composting dates to 1990.)

Material Recycling and Composting
Percent of Total Waster Generation

Year	**Compost**	**Recyle**	**Total Recovery**
1950		1.0%	1.0%
1960		6.4%	6.4%
1970		6.6%	6.6%
1980		9.6%	9.6%
1990	2.0%	14.2%	16.2%
1995	4.5%	21.5%	26.0%
1998	5.9%	21.5%	27.4%
1999	6.4%	21.7%	28.1%
2000	7.1%	23.0%	30.1%

Complete the following tasks:

a. Develop two models (polynomial and exponential) for the Total Recovery data.

b. Discuss which is the better model (do more than just noting the R^2 values).

c. Predict the percentage of waste composted, recycled, and totally recovered in 2010.

d. Form an exponential regression for Total Recovery when the Year data is entered as 0, 10, 20, 30, 40, 45, 48, 49, 50. Then shift the resulting regression given formula 1950 units to the right. Compare the result to the regression formula obtained when the year data is entered as 1950, 1960, 1970, ... Comment on the comparison.

Visit our website:
www.ContemporaryCollegeAlgebra.org

[5] Notices

1. Fall Retreat for instructors interested in teaching Contemporary College Algebra will be held at Cy-Fair College in northwestern Houston, TX, October 9-11, 2003. Those interested in attending please contact Laurette Foster (Laurette_Foster@pvamu.edu) or Don Small (don-small@usma.edu). The program will begin with an informal supper Thursday night and conclude with lunch on Saturday. A grant from the Brown Foundation provides financial support for travel, room, and board for the participants.

2. Discussions of the Contemporary College Algebra program will be held this Fall at

   a. ICTCM, Chicago, IL, Oct. 30-Nov. 2, 2003. The preconference meeting is devoted to refocusing college algebra. Don Small will help lead the preconference meeting.

   b. AMATYC, Salt Lake City, UT, Nov. 20-23, 2003. Russ Lundgren, Grace Wood, Bob Johnke, Diana Hooker, Don Small will make presentations.

3. Deadline for contributions to the November Newsletter is Monday, November 3, 2003. Send opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. to Don Small (don-small@usma.edu).

4. To subscribe to this Newsletter, write to Don Small, Dept. of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.