

# Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium\*  
Number 46, February 2003

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1) Verbally: Aspirin is quickly absorbed into the bloodstream and is then eliminated through the kidneys. Approximately half the aspirin is eliminated within the first three-fourths of an hour and most of it within three hours.

2) Numerically:

Time (min)	0	15	30	45	60
Aspirin (gm)	0	.082	.071	.053	.037
Time (min)	75	90	105	120	150
Aspirin (gm)	.025	.017	.011	.007	.003

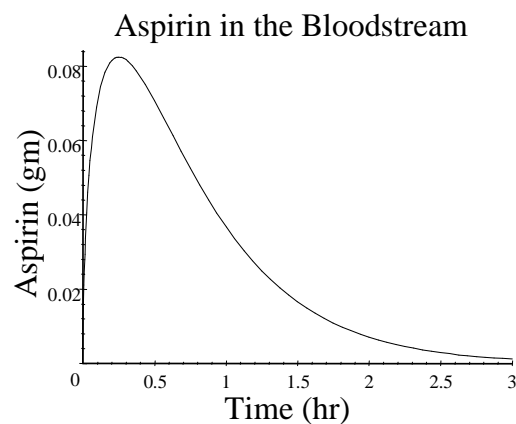
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## [1] Presenting Functions

Functions are commonly presented in four ways: 1) verbally (describe in words), 2) numerically (table of values), 3) graphically, 4) symbolically (formula). The verbal description is primarily used when a qualitative description is desired while the other three are used for quantitative descriptions. We illustrate with a function modeling the absorption and elimination of aspirin in a person's bloodstream. (Because absorption and elimination rates vary from person to person, the rate in this model is understood to be an approximation.)

\* Supported by the National Science Foundation and the U.S. Military Academy.

3) Graphically



4) Symbolically:  $f(t) = 0.1\sqrt{t}e^{(1-2t)}$  where  $t$  is time in minutes and  $f(t)$  is the amount (mgs) of aspirin in the bloodstream  $t$  minutes after an aspirin pill is swallowed. (Assume there is no aspirin in the blood system at time

zero.)

The choice of presentation usually depends on the purpose and the audience. The following list contains many of the important purposes of presenting a function. For each one, determine the most appropriate method of presentation.

1. (a) Approximating when the amount of aspirin in the bloodstream is a maximum.
- (b) Approximating the amount of aspirin in the bloodstream after 2 hours.
- (c) Describing to a patient how fast aspirin “works.”
- (d) Comparing the rate of absorption to the rate of elimination.
- (e) Approximating how long it takes to eliminate three-fourths of the aspirin.
- (f) Sampling the amount of aspirin in the bloodstream for different times.
- (g) Modifying the model.

Note how the graphically model changes when you What-if the symbolic model (e.g., change the coefficients, change the power).

## [2] Test Questions

a. The local pizza shop sells two sizes of pizza, a large 12 inch diameter pizza for \$10 and a medium 8 inch diameter pizza for \$5. Both pizzas have the same thickness. If you wish to maximize the amount of pizza you can buy

for \$10, should you order a large pizza or two medium pizzas? Explain your reasoning.

b. Sarah’s Run. Sarah warms up for her run by jogging for 5 minutes, then runs fast for 15 minutes, then runs at a moderate pace for 20 minutes, and completes her run with a 5 minute walk. Sketch two graphs: the first showing Sarah’s velocity as a function of time and the second showing the distance Sarah travels as a function of time.

c. How many zeros does  $f(x) = \frac{\cos(2x+3)+x^2-e^x}{1+x^2}$  have? Approximate the numerical value of the zero(s) and clearly explain why there are not more zeros.

d. There is a growing debate over the fairness of President Bush’s proposed \$674 billion tax plan. One side of the debate cites the average savings that will accrue to the taxpayer and the other side cites the median savings that will accrue to the taxpayer. Does the side supporting the tax plan cite the average or the median savings as an indication of fairness? Explain your reasoning.

## [3] Carry-on Luggage

Many airlines now restrict passengers to two carry-ons, one of which is a personal item (e.g., computer). Size restrictions on the other carry-on state that the sum of the length, width, and height must be less than or equal to forty-five inches. Determine the maximum volume of a legal sized carry-on that has a height of eleven inches.

Think about the ratio of the width to the length for the carry-on of maximum volume.

Is the ratio surprising? Does the ratio change when you rework the problem assuming a different height? Rework the problem a second time with a different height and check the ratio of the width to the length. What conjectures can you make about the ratio of the width to the length of a box of maximum volume having a fixed height? What conjecture can you make concerning the ratio of the sides of a rectangle having maximum area? Explain your reasoning.

#### [4] Sm. Group Activity - Buying Pizzas

Central Pizza in York, PA sells a small (8 inch diameter) pizza for \$5, a medium (12 inch diameter) pizza for \$8, and a large (16 inch diameter) pizza for \$12. All their pizzas have the same thickness. Tom can spend up to \$25 for pizzas for a party he is giving on Saturday. What should Tom order to get the most pizza for his money? Explain your reasoning.

On Sundays Central Pizza offers a \$4 reduction on each order over \$20. If Tom changed his party to Sunday, would he change his pizza order? Explain your reasoning.

#### [5] Essay Topic: The Uniqueness Property of Functions

Write a one page essay discussing the uniqueness property of functions. Some points or questions to consider are:

- What is the uniqueness property.
- How can you check to see if a relation with one independent variable satisfies the uniqueness property? (Remember to consider the different ways a function may be expressed.)

- Examples of relations that do satisfy the uniqueness property and examples of relations that do not satisfy the uniqueness property.
- How important is the uniqueness property?

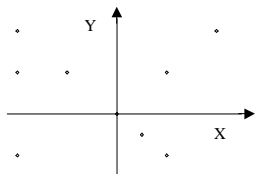
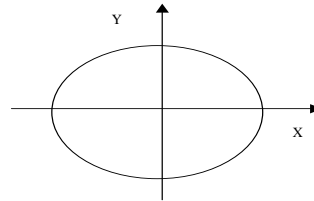
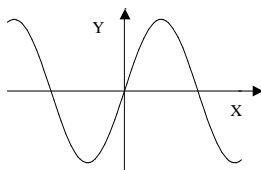
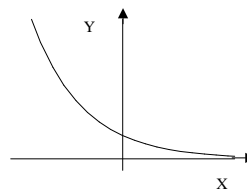
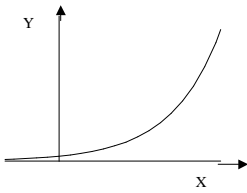
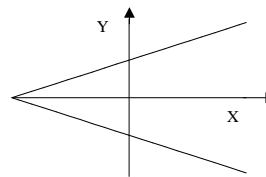
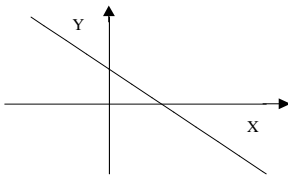
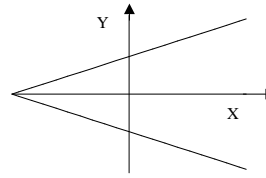
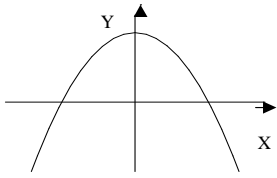
#### [6] Quickies

- Consider taking a multi-legged trip. Is the average speed for the whole trip the same as the average of the speeds over each leg? Give an example illustrating your answer.
- Consider an equilateral triangle inscribed in a circle of radius  $r$  and a square inscribed in a circle of radius  $r$ . Which one (triangle or square) contains the larger area. Explain your reasoning.
- Does the vertical line test for determining if a relation is a function apply to a relation with two independent variables? Why?

#### [7] Identifying Relations

Identify each of the following plots by writing the letter corresponding to the appropriate identification from the following list.

- $Y$  is probably a linear function of  $X$ .
- $Y$  is probably a quadratic function of  $X$ .
- $Y$  is probably a cubic function of  $X$ .
- $Y$  is not a function of  $X$ .
- $Y$  is probably an exponential function of  $X$ .
- $Y$  is probably a square root function of  $X$ .



## [8] Notices

1. Deadline for contributions to the March Newsletter is Monday, March 3, 2003. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.
2. To subscribe to this Newsletter, write to Dr. Della Bell, Chair, Department of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.