Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Average as a *Balance Point*

Average is probably the most used and probably the most important statistic associated with a data set. People speak of *slugging average* in baseball, *average sales* for the past month, *average* amount of rainfall, *average* temperature, *average* class size, *average* test score, etc. Performance in a very broad spectrum of activities is measured in comparison to an average. That is, in how much the performance varies from the average. In a numerical data set, the variation of an element, x_i, is $x_i - \overline{x}$, where $\overline{x}$ represents the average of the data set. The variation of an element can be either positive or negative depending on whether the element is greater than or less than the average.

The interpretation of average as a balance point reflects the fact that the amount of negative variation in a numerical data set equals the amount of positive variation in the data set. The following analysis of telephone rates for the 10-10-345 system illustrates average as a balance point. In a holiday special, the 10-10-345 phone company offers the following rates (per minute) in addition to a thirty cent connection fee.

Country	**Fee (cents)**	**Variation**
Brazil	25	
Hong Kong	8	
Japan	11	
Peru	32	
Poland	19	
Spain	17	

1. (a) Compute the average fee per minute of calling time.

   (b) Fill in the Variation column of the table.

   (c) Sum the column of variations.

   (d) Explain why the sum in part c has to be zero.

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(e) Illustrate your reasoning in part d by making up a numerical data set, computing and then summing the variations.

[2] Number Game

The Interactive Mathematics Program (IMP), a reformed secondary school mathematics program, emphasizes solving word problems, small group activities, discovery learning, and extended group problems. In 1999, the United States Department of Education named IMP one of the nation's top five exemplary mathematics programs. Creating number games, like the following one, is part of their program.

> Pick any number, multiply it by 2, add 8, divide by 2, and subtract the original number. The answer is 4.

Small Group Activity. Ask each group to explain why the sequence of steps in the preceding game always yields four. Then pair off the small groups and have each group make up an interesting challenge game for their partner group. After the groups have solved their challenge game, share a few of the games with the whole class. Post some of the number games on the department's bulletin board for others to play.

[3] Average, Median, Mode of a Deck of Cards

Remove the four aces from a deck of playing cards. The numerical card value for cards 2 through 10 is the face value of the card. Let the numerical value for jacks, queens, and kings be 10.

1. (a) Compute the average card value.

   (b) Is the average card value greater than the median card value?

   (c) How does the mode value compare with the average and median values?

   (d) Draw a (frequency) bar chart with the card values represented on the horizontal axis and the frequency of those values occurring represented on the vertical axis.

   (e) How can you tell the mode of a data set given a frequency bar chart?

[4] Haitien Exchange Rate

The exchange rate of United States to Haitien currency was one U.S. dollar to five Haitien dollars in January 2002. On November 27, 2002, the *New York Times* reported that "Haiti's currency, the gourde, has lost 40 percent of its value in the last year." Determine what the exchange rate should be in January 2003.

[5] Cost of Electricity

The rate schedule for Orange County, NY of the Central Hudson Power and Light Company shows the following charges in terms of kilowatt hours (kWh):. These charges include the cost of the electricity, delivery service, meter maintenance, and New York State taxes.

No. of kWh	Cost
100	\$11.90
300	\$35.71
500	\$59.50
800	\$95.20
1200	\$142.80
1500	\$178.50

1. Do the following:

1. (a) Plot the data in the table.
   (b) Fit a line to the data plot. That is, determine the equation of a line whose graph contains the data points.
   (c) Explain what the slope of the line represents in terms of this problem.
   (d) Explain what the intercept represents in terms of this problem.
   (e) Determine the cost for 1370 kWh.

2. Describe how the line in part 1b would change under each of the following scenarios. Explain your reasoning.

1. (a) The price per kWh increases as a result of increasing generating costs.
   (b) The price of meter maintenance increased by 50%.
   (c) New York State decreased its taxes on the use of electricity.

[6] Displaying Soup Cans

A display of soup cans forms a triangular shape with several rows of cans placed on top of one another according to the restriction that each row, counting from the top, has the same number of cans as its row number. Two natural questions are 1) how many cans are in a display having n rows? And 2) how large a display (number of rows) can be formed from k cans?

Experiment by counting the number of cans in a display having 1, 2, 3, rows to gain an understanding of the problem

Consider mentally constructing a display from the top down one row at a time. That is, begin with one row (one can), then add a second row, then a third row, etc. Note the number of cans in a 2-row display is the number required for a 1-row display plus 2 and the number of cans in a 3-row display is the number required for a 2-row display plus 3 and so on.

Form a 2-column table listing the number of rows and the corresponding number of cans.

No. of Rows	No. of Cans
1	1
2	3
3	6
4	10

Develop a recursive sequence model for the number of cans in an n-row display.

Let c(n) = number of cans needed to form an n-row display..

c(n) = c(n-1) + n

c(0) = 0 (zero row, zero sum, zero display)

Discover a formula for c(n) and then evaluate c(10).

Hint: Plot the data from the table (extend the table to n=10), recognize the basic shape of the data, and then graphically fit a curve to the data or use the suitable regression program in your calculator.

An Alternative Approach. Express c(n) as the sum of the seats in n rows rather than as a recursive sequence. That is,

No. of Rows	**No. of Cans**
1	c(1)=1
2	c(2)=1+2
3	c(3)=1+2+3
4	c(4)=1+2+3+4

Karl Friedrich Gauss (1777-1855), a famous German mathematician, developed a creative way of summing the first n positive integers when he was a nine year old schoolboy. One day his teacher, hoping to keep the students busy for awhile, assigned them the task of summing the first 100 positive integers. Almost immediately Gauss wrote the correct answer (5,050) on his paper. How did he compute the sum? Certainly not by adding the 100 numbers one at a time.

Gauss' method was to (mentally) write down the 100 integers, in increasing order, in a row. On the second row he wrote down the 100 integers in decreasing order. He then added each number in the second row to the corresponding number in the first row, getting 101 each time.

$$\begin{array}{l} 1+ \quad 2+ \quad 3+\ .\ .\ .\ +100 \\ \underline{100+\ \ 99+\ \ 98+\ .\ .\ .\ +} \quad 1 \\ 101+101+101+.....+101 \end{array}$$

Gauss now had 100 sums of 101 or 10,100. Since each integer had been counted twice (once in the first row and once in the second row), the 10,100 represented two times the sum of the first 100 positive integers. Thus

$$\begin{aligned} C(100) &= 1+2+3++100 \\ &= \tfrac{(100)(101)}{2} \\ &= 5,050. \end{aligned}$$

Using Gauss' method, compute c(12).
Generalize Gauss' method, to obtain a formula for c(n).
Superimpose the plot of c(n) on the data plot formed from the table of values.

[6] Notices

1. The Contemporary College Algebra program will be well represented at the Joint Mathematics Meetings to be held in Baltimore, MD, January 15-18, 2003. Dorothy Hunter will conduct a Poster Session on "First College-Level Mathematics Course;" Alex Fluellen will speak on the panel "Reflections on the Conference to Improve College Algebra;" Paul Dirks and Laurette Foster will speak on the panel "Small Group Projects in College Algebra." As members of CRAFTY, Laurette Foster and Don Small will be deeply engaged in developing MAA policy towards refocusing college algebra on data, functions, and modeling.

2. Deadline for contributions to the February Newsletter is Monday, February 3, 2003. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

3. To subscribe to this Newsletter, write to Dr. Della Bell, Chair, Department of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.