Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Cheese: Buy in Bulk?

Jim and Bianca run a Deli in their Central Pizza Shop in York, PA. Some of their vendors offer discounts on bulk purchases. On the surface, these offers appear to be money saving propositions. However these savings sometimes vanish, when the cost of the extra financial investment as well as storage are computed. A recent example involved buying imported Italian cheese: Parmesan Reggiano. Jim usually sells 5 pounds of this cheese per month. He normally pays \$6.60/lb and sells it for \$11.95/lb. The salesman offered to sell Jim half a wheel of the cheese (30 lbs) for \$6.00/lb. Would Jim make money buying the 30 lb block of Parmesan Reggiano?

Assume that Jim can borrow money at 6% APR, cooler space for the extra cheese would average one dollar per month, and that other additional business expense (e.g., insurance) would be compensated by having a supply that could possibly satisfy an unexpected demand.

Compute Jim's monthly cost on the investment associated with buying the 30 lb block of cheese. This cost will be different each month. Why? Sum the monthly amounts along with the cooler cost and then subtract that from the *apparent* savings. Does Jim realize a savings from buying the 30 lb block instead of a 5 lb piece each month? How much is his saving or loss?

Hint: Assume Jim buys the 30 lb block. During the first month he is paying for 25 lbs of cheese that he will not sell. Thus he needs to borrow 25 x \$6.00 = \$150 at 6% APR for 6 months. What is the situation for the second month?

[2] Simulating Car Rentals

Earl has purchased a 3-city *Rental All* franchise, a car rental agency. He has 300 cars to distribute

* Supported by the National Science Foundation and the U.S. Military Academy.

between the three cities, but does not know how many he should assign to each of the cities. Furthermore the daily records from the previous owner are confusing to him. They indicate that on an average day: 30% of the cars in city A end up in city B, 10% end up in city C, and the rest remain in city A. For city B, the results are that 15% of the cars end up in city A, 20% end up in city C, and 65% stay in city B. For city C, 20% of the cars end up in city A, 29% end up in city B, and the remainder stay in city C.

Your task is to two-fold.

1. Provide Earl with a diagram model of the daily movement of the rental cars between the three cities.

2. Run a simulation of the daily movement of cars between the three cities for several consecutive days. Continue the simulation until the long term distribution of the cars is established.

Hint for task 1: Complete the following *diagram.*

Hint for task 2: Define a recursive sequence for each of the three cities. For example: Let

> $a(n)$ be the number of rental cars available at City A on day n.
> $b(n)$ be the number of rental cars available at City B on day n.
> $c(n)$ be the number of rental cars available at City C on day n.

Let $a(n+1)$ be the the number of cars that stayed in city A during day n plus those which came from city B plus those which came from city C. Thus

$$a(n+1) = .60a(n) + .15b(n) + .20c(n)$$

Form the corresponding recursive sequences for $b(n+1)$ and $c(n+1)$.

Express the system of three recursive sequences as a matrix equation in the form

$$\begin{bmatrix} a(n+1) \\ b(n+1) \\ c(n+1) \end{bmatrix} = \begin{bmatrix} .60 & .15 & .20 \\ - & - & - \\ - & - & - \end{bmatrix} x \begin{bmatrix} a(n) \\ b(n) \\ c(n) \end{bmatrix}$$

Begin your simulation with 100 cars in each city, that is set $a(0) = 100, b(0) = 100, c(0) = 100$. Completing the matrix multiplication gives the distribution after one day.

The computations can be done very easily using a graphing calculator. For example, define [A] to be the 3 x 3 coefficient matrix and define [B] to be the 3 x 1 matrix of initial values. Then [A] $\cdot$ [B] gives the distribution of cars after day one. [A] $\cdot$ [A] $\cdot$ [B] = $[A]^2 \cdot$ [B] gives the distribution after two days, etc. with $[A]^n \cdot$ [B] being the distribution after n days.

How do you advise Earl to distribute his cars?

As an additional task, see what happens if your initial distribution of the 300 cars is not equal between the cities. What prediction can you make about the long term distribution.

[3] The Length of Ups and Downs

What do bouncing tennis balls, vibrating drum heads, and bungee jumpers have in common? Answer: They all share the experience of oscillatory motion that *damps out* over time. In this problem, you are asked to determine how far a bungee jumper travels as well as determining her final position.

Suppose Margaret plunges off a bridge with her feet firmly attached to a bungee cord that is also attached to the bridge. Assume her initial plunge carries her 150 feet below the bridge. Also assume the length of each rebound is 60% of the length of the preceding drop and the length of the following drop is 40% of the length of the preceding rebound. Determine how far Margaret travels on her bungee ride and how far below the bridge is her final position.

Hint: Express the sum of the lengths for several (say, 8) *down-up* cycles for the purpose of recognizing a pattern. Be alert to the possibility that the sum of the odd terms might represent a different pattern than the sum of the even terms. Could one expect a geometric series situation? Why? (Remember, when looking for a pattern, do not carry out any addition or multiplication operations that could disguise the pattern. However, expressing repeated multiplication using exponential notation is encouraged.)

[4] The Price of Envelopes

The *Viking* April Sale catalogue for business office supplies showed the cost of imprinted business envelopes depended on the number of boxes purchased. Each box contains 500 envelopes. The price scale per box was

$2 boxes$	$5 boxes$	$10 boxes$	$20 boxes$	$50 boxes$
\$21.49	\$18.49	\$16.49	\$14.49	\$12.99

Plot this data and then graphically fit a curve to the data. Use the resulting function to determine a reasonable price for 13 boxes of envelopes.

[5] "Quickies"

a. Sandra is eligible to receive \$1,642 per month from Social Security. However, if Sandra waits for three and one-half years until she is 70, her payments will be \$1,977 per month. If Sandra begins accepting her Social Security payments now, how many years will it be before the total amount she receives would be equal to the amount she would have received if she had waited until she was 70 before starting payments?

b. Is it better to get a 10% discount on an item whose price has already been discounted 30% or to get a 40% discount on the original price? Explain.

c. Express the number of diagonals in a convex polygon as a function of the number of vertices in the polygon. (A diagonal is determined by any two non-adjacent vertices.) Hint: Draw some pictures and look for a pattern by creating a table showing the number of vertices and the number of diagonals for an n-gon for consecutive values of n beginning with $n = 3$.

[6] Riding a Ferris Wheel

(This problem is adapted from exercise number 9 on page 198 of *Contemporary Precalculus through Applications: functions, data analysis and matrices* by Gloria B. Barrett and others at the North Carolina School of Science and Mathematics, Jason Publications, Inc., Dedham, MA, 1992.)

The amusement park has a ferris wheel that is 32 feet in radius. The lowest point on the wheel is 4 feet above the ground and the wheel rotates once every 15 seconds. Model, graphically and analytically, the height from the ground of a person riding the ferris wheel.

[7] Aspirin Dosage

Don's doctor has advised him to take a baby aspirin (81 mg) each day to thin his blood and prevent a heart attack. While on an extended camping trip, Don used up all of his baby aspirin tablets. Although he had some regular strength aspirin (324 mg) tablets with him, he did not know how often to take them in order to maintain the doctor's *recommended aspirin level* in his system. Knowing that aspirin overdose can cause ulcers, bleeding, etc., he decided to ask for help to investigate the long term situation rather than assume the simplistic answer of one 324 mg tablet every four days.

Please provide the requested help by

a. Creating a recursive model for the *aspirin level* in Don's system resulting from taking a baby aspirin each day. Assume that Don's body eliminates 20% of the aspirin in his system each day.

b. Determining the long term *aspirin level* from the model in part (a).

c. Creating a recursive model for the *aspirin level* in Don's system resulting from taking a regular aspirin tablet every four days. What is the long term behavior of this model?

d. If the long term behavior of the model in part (c) differs from that in part (b), redo part (c) increasing the period (5 days, 6 days, etc.) until the long term behavior is similar to that in part (b).

Appreciate how easy it is to modify the model in part (a) to represent different dosage levels and different time periods as well as different elimination rates. Also appreciate the *power* of mathematical modeling in that the model in part (a) is basically the same model used for determining the balance in a savings account, the balance due on a car loan, the temperature of a warming can of soda, concentration level when diluting a brine solution, etc.

[8]

Notices

1. The fourth edition of *Contemporary College Algebra* by Don Small is now available (ISBN: 0-07-256439-3). Examination copies may be obtained by contacting the McGraw-Hill Publishing Co (1-800-338-3987).

2. The next issue of the *Vision - Potential* Newsletter will appear in September 2002. The <u>Deadline</u> for contributions to the September Newsletter is <u>Monday, September 2, 2002</u>.

   Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are welcomed. Please send material to Dr. Della Bell, Chair, Department of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

3. To subscribe to this Newsletter write to Dr. Della Bell, Chair, Department of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.