

Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

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[1] HBCU College Algebra Reform

Major changes in undergraduate education have taken place in the past few years. The increased application of mathematical modeling and reasoning in several quantitative disciplines is leading to inter-disciplinary cooperation between the faculties in mathematics and partner disciplines. The pedagogical changes arising from the Calculus Reform Movement has shifted the focus from instructor presentation to student engagement in his or her own learning. Providing small group learning

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opportunities, developing communication skills, developing an inquisitive and exploratory approach to mathematics, and learning how to learn are now commonly listed as course objectives. The past few years has also seen an explosion in the development of computing and graphing technology and their use in learning and doing mathematics.

In spite of the major changes taking place in undergraduate mathematics education, the content and pedagogy of College Algebra courses has not changed significantly. At present (1) Instructional emphasis is almost solely on symbolic manipulation; (2) There is little to no use of technology; (3) Lecture is the predominant format for instruction; (4) Problem solving and critical thinking skills are not adequately addressed; (5) Little or no emphasis is placed on inter-disciplinary applications; (6) Development of communication skills is not addressed; (7) Upon leaving the course, student preparation is often inadequate for follow-along courses and work responsibilities.

Objectives (HBCU College Algebra Reform Project):
Change the culture of the College Algebra course by

- A. Changing what is taught;
- B. Changing the pedagogy; and

C. Changing the role of College Algebra within the college or university.

A. Changing what is taught

A new course will be developed to focus on analyzing data, modeling real world problems, and developing problem solving skills. Students, working in small groups, will model, solve, and interpret interdisciplinary problems developed by interdisciplinary teams of faculty members. Technology will be used to analyze situations using numeric, graphic, and symbolic representations.

B. Changing the pedagogy

Student growth models will be developed resulting in students taking a greater responsibility for their own learning and being more engaged in the learning process. In particular, small group work will be emphasized, both in terms of in-class activities and out-of-class projects. Writing assignments and class presentations will be routinely assigned. Students will use graphing calculators and/or computers for most of their assignments. Faculty will use calculator or computer projection devices and calculator based laboratory units in their classes.

C. Changing the role of College Algebra within the college or university

The “partner” disciplines (those requiring College Algebra) will be involved in the development and execution of the new college algebra program in two explicit ways

- a. Each partner discipline requiring College Algebra will be asked to detail the reasons for the requirement and asked to help formulate a College Algebra curriculum that better meets their needs.
- b. Teams consisting of faculty from mathematics and one or two partner disciplines will develop Interdisciplinary Lively Application Projects (ILAPs) which will be assigned on a group basis in the College Algebra course and referenced in the partner discipline.

The cooperation with partner disciplines will produce a College Algebra course that is a college or university course rather than just a mathematics’ department course. The course will better coordinate the mathematics taught with the needs of the partner disciplines and the partner disciplines will share responsibility for the course.

Editor’s Note. The above passage was copied from the 1996 funding proposal. Several of the issues, namely small group activities, use of technology, and writing, were incorporated into last semester courses at most of the Consortium Schools.

[2] **Vignette - George Walker**
(Written by **Lawrence Woodard**)
(Grambling State University)

George Theophilus Walker (1922) is the first Black American composer to win the prestigious Pulitzer Prize for music. The award was presented to him in 1996 for his composition “Lilacs,” a work for soprano voice and orchestra that is based on a poem by Walt Whitman. The piece premiered with the Boston Symphony Orchestra on February 1, 1996. Mr. Walker said “In ‘Lilacs’, he sought to use Whitman for his identification with Lincoln, a symbol of freedom and emancipation especially to blacks.

Mr. Walker, a leading post World War II composer and pianist, was born in Washington, D.C. in 1922 and presently resides in Montclair, NJ. He was introduced to music at an early age by his mother who, he says “started him on the piano when she heard him banging on it.” Shortly afterwards, his talent for music was recognized and he gave his first concert at age 14.

His parents, who greatly valued education, strongly encouraged and supported his musical efforts. Mr. Walker’s father, a Jamaican immigrant, put himself through medical school and formed medical clubs to do research when barred by the American Medical Association because of his race. His

mother tutored neighborhood children in math and writing while raising George and his sister. Mr. Walker's education was obtained at Oberlin College (B.Mus, 1941); the Curtis Institute of Music in Philadelphia, as a student of Rudolf Serkin, (Artist Diploma, 1945); the American Academy at Fontainebleau, France where he studied under the famed Nadia Boulanger (Artist Diploma, 1947); and at the Eastman School of Music (D.M.A., 1947).

In recognition of his musical talent and skills, Mr. Walker has been the recipient of various scholarships and awards from Oberlin College, Eastman School of Music, Rockefeller Foundation, and the Guggenheim Foundation, to name a few. In 1945, as winner of the Philadelphia Youth auditions, he performed with the Philadelphia Orchestra. In the same year, he made his debut at Town Hall in New York.

Mr. Walker's ambition was to have a career as a concert pianist. But he faced a major obstacle as a Black aspiring pianist in this country in the 1950s. For several years, he was unable to secure sponsorships, however, he persisted and in 1953 National Concert Artists booked him for a European tour. Of this experience, he remarked that "he never got the opportunities that would have allowed him to concertize like a white pianist." Although discouraged and disappointed, he prevailed and refused to harbor resentment for this situation, and in his own words "I never felt bitter. I strongly felt that if I continued to press for what I hoped to achieve, I would achieve it."

In the education field, Mr. Walker has held teaching positions at Smith College in Massachusetts (1961-69), University of Colorado (1961-69), and Rutgers University (1969-92), where he served as Chairman of the Music Department.

After 60 years of composing, performing, and teaching, his persistence, talent, and skill as a composer have assured him a place in history as the first Black American to receive the Pulitzer Prize for music. His more than 70 published works in-

clude sonatas, string quartets, cantatas, overtures, concertos, and a Mass. His music has been described as quite complex and technically challenging with a propensity for classical forms and the use of contemporary elements. Strains of jazz and other Afro-American idioms also permeate portions of his music.

[3]

**Watering Horses
Tong Wu
Texas Southern University**

(This is a small group, in-class activity that involves applications of additions and subtractions and imagination.)

Problem There are three containers. The first container's volume is fourteen gallons. The second container's volume is three gallons. The third container's volume is five gallons. The first container is full of water and the second and third containers are empty. There are four horses to water. Each horse needs respectively one gallon, two gallons, four gallons, and seven gallons of water. How can you give the correct amount of water to each horse? (Note: the containers do not have any volume markings on them.)

[4]

Credit Card Debt

(This is a small group, in-class modeling activity that has several real life applications.)

You have just received the monthly notice from your credit card company and must now deal with the financial realities of last New Year Eve's Party spree. Your debt is \$2,000. Your credit card company, as required under the "Truth in Lending Act," publicizes its Annual Percentage Rate (APR) to be 14%. This is the percentage used when compounding the interest yearly. However, your credit card company compounds interest monthly and thus your monthly interest rate is

$$\text{monthly rate} = \frac{APR}{12} = \frac{0.14}{12} = 0.0117$$

rounded to 4 decimal place accuracy. An additional consideration is that your credit card company requires a minimum payment of \$25/month.

Analyze your situation by answering the following questions assuming you do not make any new charges to your account.

1. Write a mathematical model for the unpaid monthly balance. Clearly define all variables. (Let p denote the monthly payment.)
2. How many months will be required to pay off your debt if you pay just the minimum each month? Explain your answer.
3. How many months will be required to pay off your debt if you pay \$40 each month? If you pay \$50 each month?
4. How large would your monthly payment have to be in order to pay off your debt in 12 months?

[5] **A Writing Assignment**

Writing Assignment: Write a description of the following problem and solution in which you explain the logic in the solution to your younger brother or sister.

Problem Josh wants to buy several large crunchy chocolate bars. He can buy them for 50 cents apiece or two for ninety cents or three for one dollar and twenty cents. If Josh has \$3.50 to spend on candy, how many large crunch chocolate bars can he buy? What should be his selection?

Solution: The first step in problem solving is to define the variables. Thus let

1. x = number of sets of 3 bars
2. y = number of sets of 2 bars
3. z = number of single bars

Number of candy bars purchased = $3x + 2y + z$
 Cost of candy bars purchased = $1.20x + .90y + .50z$

Question: How to maximize the number of bars purchased subject to the cost constraints? That is,

1. How to maximize $3x + 2y + z$ subject to $1.20x + .90y + .50z < 3.50$?

Solution Approach: Develop a table of possibilities

x	y	z	Cost (\leq \$3.50)	No. of Bars
0	0	7	3.50	7
0	1	5	3.40	6
0	2	3	3.30	7
0	3	1	3.20	7
1	0	4	3.20	7
1	1	2	3.10	7
1	2	1	3.50	8
2	0	2	3.40	8
2	1	0	3.30	7

Answer:

1. Josh can buy 8 large crunch chocolate bars. His best selection is to buy two triple sets and two single bars.

[6] **Activity: Outrunning a Train**

(This is a small group, in-class activity that emphasizes the importance of clearly defining variables when modeling a situation.)

Jake finds himself $3/8$ of the way across a railroad bridge when he hears an approaching train. Assume Jake can run 15 mph and also assume that if he runs to either end of the bridge, he and the train will arrive at that end of the bridge at the same time.

Analyze the situation by answering the following questions.

- Draw and label a picture showing the location of the train, bridge, and Jake.
- Which end of the bridge will the train reach first? (The end closest to Jake or the end furthest from Jake?) Explain your reasoning.
- How fast is the train moving?

Hints:

- $\text{rate} = \frac{\text{distance}}{\text{time}}$ or $\text{distance} = (\text{rate})(\text{time})$.
- Model the situation
 - Identify variables (e.g., length of bridge, distance of train from the bridge, time to run to each end of the bridge, speed of the train).
 - Express the times it takes Jake to run to each end of the bridge in terms of the length of the bridge.
 - Formulate 2 equations showing how far the train travels in the time it takes Jake to run to each end of the bridge.

greatly increases the value of the project. Showing a physical model of the game helps the students to understand the game.)

According to legend the Tower of Hanoi consisted of 64 disks of varying diameters. The disks were stacked so that no larger disk was on top of a smaller disk. The tower was attended by a mystic order of monks who were charged with the task of taking the tower apart and rebuilding it on another site with a third site available for “intermediate” storage of disks. The long established ritual for moving the disks was to move one disk each minute and to never place a larger disk on top of a smaller disk. The monks believed that as soon as the tower could be reassembled with all 64 disks, the earth would collapse in a cloud of dust.

The French mathematician Edouard Lucas (1842-1891) developed the Tower of Hanoi game based on the above legend. His game consists of a board with 3 pegs and 7 disks of different diameters. The game begins with all 7 disks on one peg, with the largest disk on bottom and the smallest disk on top (see figure 1.).

[7] Query

Suppose you drive from Austin to Marshall, TX. Assume that you start with a full tank of fuel and purchase 8 gallons of fuel on the way. If you arrive in Marshall with $1/5$ of a tank of fuel, how many gallons does your fuel tank hold?

[8] Activity: “Tower of Hanoi”

(This is a “hands-on” small group in-class activity or out-of-class project designed to challenge students to discover a (recursive) pattern for playing the game. When used as an out-of-class project, requiring each group to submit a written report describing the group’s reasoning (and false attempts)

The object of the game is to move all of the disks, one at a time, in the smallest possible number of moves, to the third peg to form an identical “tower.” During the moves, the player is not allowed to place a larger disk on top of a smaller disk? Can the objective be achieved? If so, what is the minimum number of moves required? How many moves would be required to move 64 disks? How long will it take the monks to move all 64

disks?

Hint: Encourage each group of students to make up a physical model (coins, books, pieces of paper, etc) with which they can play the game and implement the following discovery approach

- a. Generate data
- b. Conjecture a pattern
- c. Verify your conjecture

The first step, generate data, is done by playing the game several times. First with 1 disk, then with 2 disks, then with 3 disks, etc. Write out a (uniform) description of the steps in the solution of each of these games as well as an explicit relation between the number of disks and the number of moves. (Let $m(n)$ = the minimum number of moves required to move n disks. So $m(1) = 1$, $m(2) = 3$, etc.)

The second step, conjecture a pattern, can be accomplished in at least two ways. One is to look at the written descriptions of the solutions obtained in step one and ask "What changes are required when I include just one more disk?" Another is to study the sequence of numbers $m(1)$, $m(2)$, $m(3)$, $m(4)$,

The third step, verify your conjecture, is done by playing the game with a larger number of disks than were used in step one and comparing the actual number of moves against the number given by your conjecture.

[9] Notes and Announcements

1. The Deadline for contributions to our February Newsletter is
Wednesday, February 12, 1997.
2. Calculus Reform Workshop, 20-22 February 1997. The Workshop is sponsored by Paul Quinn College and the National Science Foundation (NSF). Participants will receive their choice of one of the following: TI calculator, Math T/L software, Derive software, or (student) Maple software. All participant expenses are paid by the NSF except for personal travel. Sarah Bush and Don Small will be the instructors. Contact Dr. Najarajan (214-302-3625) for details.
3. Please post the list of 1997 NSF Calculus Reform Workshops and encourage your colleagues to consider attending one of the workshops. All participant expenses are paid by the NSF except for personal travel.

WANTED for our Newsletter: Opinion articles, suggestions for writing assignments, small group in-class activities, small group out of class projects, Quick Questions, CBL activities, announcements, etc. Please send material to Della Bell, Dept. of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.