Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

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[1] Towers of Hanoi

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The Legend of the Towers of Hanoi

> In an ancient city, so the legend goes, monks in a temple had to move a pile of 64 sacred disks from one location to another. The disks were fragile; only one could be carried at a time. A disk could not be placed on top of a smaller, less valuable disk. In addition, there was only one other location in the temple (besides the original and destination locations) sacred enough for a pile of disks to be placed there.

Using the intermediate location, the monks began to move disks back and forth from the original pile to the pile at the new location, always keeping the piles in order (largest on the bottom, smallest on the top). According to the legend, before the monks could make the final move to complete the new pile in the new location, the temple would turn to dust and the world would end.

Is there any truth to this legend?

Before answering, do the following in order to determine how long it will take the monks to finish their task.

a. Play the game, with three disks, to determine the smallest number of moves necessary to move all of the disks.

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b. Repeat the game with four disks.

c. Continue playing the game with larger numbers of disks until you discover a mathematical pattern for the minimum number of moves necessary to move n disks.

d. Write a short paragraph justifying your pattern by clearly explaining your reasoning.

e. Use your pattern to determine the minimum number of moves required to move 64 disks.

f. Assume it takes one second, on average, to move a disk. How many years would it take the monks to move all 64 disks?

Now you are prepared to answer the question (along with an explanation for your answer).

[2] Income, Taxes, Charity

In a piece in the *New York Times*, February 18, 2001, the following data appeared under the title "How Itemized Deductions Average Out by Income Groups." The entries for State and Local Taxes and for Charitable Giving are averages for the amounts reported on the 1998 tax returns. In order to help gain an understanding of these figures, do the following:

1. Fill in the last column with the percentage of the Adjusted Income that is given to Charity. Write a paragraph commenting on the percentage of charitable giving as a function of adjusted income.

b. Plot the data for Local and State Taxes as a function of Adjusted Income and then graphically fit a curve to the data.

c. Plot the data for Charitable Giving as a function of Adjusted Income and then graphically fit a curve to the data.

Adjusted Income (thousands)	*State and Local Taxes (hundreds)*	*Charitable Giving (hundreds)*	*Percent*
15	20.40	15.06	
20	21.55	15.43	
25	22.76	16.62	
30	26.36	16.00	
35	28.43	16.25	
40	30.90	16.78	
45	34.55	16.13	
50	37.98	18.63	
55	40.61	19.92	
60	46.67	20.90	
75	60.05	24.23	
100	93.75	36.23	
200	207.44	85.70	
500	475.64	207.65	
1,000	1,820.77	1,393.80	

[3] Interpreting Plots

The plot of a decreasing function is characterized by a falling curve from upper left to lower right. Falling curves, however, can follow different paths as the following four plots show. Your task is to match up the following scenarios with their plots and then to make up a scenaro for the unmatched plot.

A. A warm can of soda placed in a refrigerator. (Temperature on the vertical axis and time on the horizontal axis.)

B. The balance due on a car loan. (Balance due measured on the vertical axis and time on the horizontal axis.)

C. The volume of coffee in a cup left on a table to evaporate. (Volume measured on the vertical axis and time on the horizontal axis.)

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[4] **Costs of Health Care**

The following data on the percentage of the U.S. gross domestic product spent on health care over the period from 1960 to 1995 was given in the paper "Building the Next Generation of Healthy people" (*Public Health Reports* (1999): 213-216). Display this data in a bar chart and in a scatter plot. Then graphically fit a curve to the data and estimate the percentage of the U.S. gross domestic product spent on health care in 2000.

Year	Percent Spent on Health Care
1960	5.1
1965	5.7
1970	7.1
1975	8.0
1980	8.9
1985	10.3
1990	12.2
1995	13.7

Hint: Sometimes an effective approach to graphically fitting a curve to a data plot is to first, recognize the basic shape of the data plot; second, shift and scale the graph of the function associated with the basic shape until a "ball park acceptable" fit is obtained; and then thirdly, "fine tune" the process by including a simpler function. (For example, a linear function is a simpler function than an exponential.) In this problem, try to obtain an exponential function whose graph generally fits the data and then fine tune the graph fitting process by adding a linear term (that is, a first degree polynomial).

[5] **A "Wet" Experiment**

As any deep-sea diver knows, water pressure increases with depth. Fluid pressure also increases as the density of the fluid increases. A diver who is fifty feet down in Utah's Great Salt Lake (salt water) will feel more pressure than if she were fifty feet deep in Maine's Katahdin Lake (fresh water). Physicists relate fluid pressure to density, gravity, and depth by the formula

$$\text{Pressure} = (\text{density})(\text{force of gravity})(\text{depth})$$

or

$$P = kgd$$

where k represents density of the fluid, g is the force of gravity, and d is depth.

In the following experiment, consider the density of water and the force of gravity both to be constant. Thus the preceeding formula says that water pressure is proportional to the depth, that is

$$\text{Pressure} = (\text{constant})(\text{depth})$$

where the constant = (density)(force of gravity). Query: What is the shape of the graph of pressure as a function of depth? Can you determine the proportionality constant from the graph? How? Consider a cylindrical water container with a hole in the bottom. Fill the container with water and watch the water squirt out of the hole. The rate at which the water squirts out is a function of the water pressure. Thus it is a function of the depth of the water in the container. Can you observe a difference in the rate that the water squirts out as the depth of the water in the container decreases? Now, one more relationship. The time required for a fixed amount of water to leak out of the container depends on the "squirt" rate, which depends on the pressure, which depends on the depth of the water. Thus there is a relation between the leaking time and depth of water. That is, it will take a shorter time for one cubic inch of water to leak out when the container is nearly full than it will when the container is nearly empty. Why?

What is the shape of the graph of leaking time as a function of depth of water?

Experimentally develop a "leaking time" function. Equipment needed: cylindrical water container (e.g., half-gallon plastic milk jug), ruler, tape, watch that will show seconds, water, and a second container to catch the leak.

Poke a small hole near the bottom of the container. Tape the ruler to the side of the container (vertically). Collect depth-time data: fill the container with water and record the time (seconds) as the water level passes each inch marking on the ruler. Make a scatter plot of the data and then fit a curve to the data set.

Use your leaking time function to determine how long it will take for the water to leak out if you started with a depth of five inches. How can you check your result?

An important principle of fluid pressure is that at any point in the fluid, the pressure is the same in all directions. (A deep-sea diver feels the same pressure on her nose and both ears.) Does this mean that the water pressure in a gallon milk jug is the same as in a half-gallon milk jug for the same depth of water?

Query: If a gallon milk jug had the same depth of water as a half-gallon jug and both had the same size hole in the same location, would it take twice as long for the gallon jug to empty as a half-gallon? Explain your reasoning.

[6] A Writing Assignment

Write a one page essay on the topic "Slope of a Line as a Rate of Change." Consider several different scenarios. For example, the slope of a moving van's loading ramp or the slope of a roof or the slope of a depreciation line or the proportionality constant in the expression that water pressure is proportional to water depth or ....

[7] Notices

1. The National Visiting Committee (NSF) for the HBCU Consortium for College Algebra Reform will meet April 27-28, 2001.

2. A writing Workshop for the HBCU Consortium for College Algebra Reform will meet April 28, 2001 at Texas Southern University.

3. The Third Edition of the *Contemporary College Algebra* text is now available from McGraw-Hill Publishing Co. Examination copies can be obtained by contacting: Regional Sales Office, Glencoe Sales Division/McGraw-Hill, Woodland Hills, California 91367 or calling 1-800-423-9534. A Teacher's Guide can be obtained by contacting Don Small don-small@usma.edu.

4. The <u>Deadline</u> for contributions to the April Newsletter is <u>Monday, April 1, 2001.</u>

5. To subscribe to this Newsletter or to submit articles, write to Dr. Della Bell, Chair, Dept. of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

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Imagination is more important than knowledge.
Einstein