Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

Number 27, April 2000

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[1] An Inquiry Project: The Value of a Dollar

What is a dollar worth? A century ago that question could be answered in terms of the Gold Standard Act, which the United States Congress passed in 1900. That Act specified that a dollar could be converted into a fixed quantity of fine gold. (The amount of gold was fixed by the Act.) Thus the value of a dollar fluctuated as the price of gold fluctuated. In 1933, the United States replaced the Gold Standard with a Modified Gold Standard. Under this Standard gold coins could no longer be circulated and citzens were prohibited from holding gold coinage. Foreign governments were still allowed to exchange dollars for gold at approximately $35 per ounce. In 1971, President Nixon "closed the gold window," totally disconnecting our currency from the value of gold. Today, the "value" of the dollar "floats" in the market place. Thus inflation, rather than the price of gold, is the major factor determining the value of the dollar. The following picture appeared in the December 20, 1999 New York Times under the heading "A Dollar Bill Won't Buy What it Once Did." (Source: Bureau of Labor Statistics)

The picture is based on the Consumer Price Index. It shows that a 1999 dollar is worth about as much as a nickle was worth in 1900.

Inquiry Project (small group)

Write a report explaining the political and/or economic climate that existed during the major changes in the value of the dollar. For example, what caused the steep drop between 1915 and 1920? Or, what caused the steep rise in the early 1930s?

* Supported by the EXXON Education Foundation and the U.S. Military Academy.

Include in your report an inflation rate chart covering the period 1900 to 1999. How does this chart compare to the New York Times picture?
Hint: Search the web (inflation, gold standard); talk with an economics professor; talk with a history professor; look up "gold standard" in the library.
(References: http://encarta.msn.com)

[2] Discovery Exercises: Fibonacci Sequence

Leonardo of Pisa (ca. 1180-1250) was the greatest Italian mathematician and possibly the greatest European mathematician in the Midle Ages. He called himself Fibonacci, a shortened version of filius Bonacci which means son of Bonacci. His book, *Liber abbaci* (Book of Calculating) published in the early 1200s, is credited with persuading many European mathematicians to change from the Roman numeral system to the new decimal system. The problem, contained in his book, that led to a recursive sequence bearing his name and for which he has become famous, states:

> A certain man put a pair of rabbits in a place surrounded by a wall. How many pairs of rabbitrs can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on becomes productive?

Thus for the first month there is just one pair, assumed to be new born. During the second month, the original pair is only one month old and therefore do not produce. Thus there is only one pair during the second month. In the third month, the original pair produces a new pair and so there are two pairs in the third month. In the fourth month, the original pair produces a new pair, but their first offspring do not as they are only one month old. Thus there are three pairs in the fourth month. In the fifth month, both the original pair and their first offspring produce giving a total of five pairs. We model the situation. Let

$u_n =$ the number of pairs of rabbits in the n^{th} month

$$u_1 = 1, \quad u_2 = 1 \quad u_3 = 2$$

The number of pairs in month n is the number of pairs that existed in the previous month, $n-1$, plus the number of new pairs. The number of new pairs is the number of pairs that existed in month $n-2$, because each of these is at least two months old and thus productive. The result is the famous recursive sequence called the Fibonacci sequence

$$u_n = u_{n-1} + u_{n-2}$$

The first ten terms of the Fibonacci sequence are

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55$$

Complete the following Discovery exercises:

1. Form the sequence $w_1, w_2, w_3, \ldots$ where $w_1 = u_1$, $w_2 = u_1 + u_3$, $w_3 = u_1 + u_3 + u_5$ and so on with w_n being the sum of the first n odd numbered terms of the Fibonacci sequence. Write down the numerical values for the first five terms of the w sequence. How are these values related to particular Fibonacci numbers?

2. Form the sequence $z_1, z_2, z_3, \ldots$ where $z_1 = u_2$, $z_2 = u_2 + u_4$, $z_3 = u_2 + u_4 + u_6$ and so on with z_n being the sum of the first n even numbered terms of the Fibonacci sequence. Write down the numerical values for the first five terms of the z sequence. How are these values related to particular Fibonacci numbers?

3. Form the sequence $q_1, q_2, q_3, \ldots$ where $q_1 = u_1^2$, $q_2 = u_1^2 + u_2^2$, $q_3 = u_1^2 + u_2^2 + u_3^2$ and so on with q_n being the sum of the squares of the first n terms of the fibonacci sequence. Write down the numerical values for the first five terms of the q sequence. How are these values related to particular Fibonacci numbers?

4. Form the sequence $p_2, p_3, p_4 \ldots$ where $p_2 = u_2^2 - u_1 u_3$, $p_3 = u_3^2 - u_2 u_4$ $p_4 = u_4^2 - u_3 u_5$ and so on with $p_n = u_n^2 - u_{n-1} u_{n+1}$. Write down the numerical values for the first five terms of the p sequence. What pattern do you recognize? What is the value of p_{20}?

[3] Summing Cubes in A.D. 1000

In approximately A.D. 1010, Abu Bekr Mohammed ibn Alhusian Alkarchi published the following method for summing the cubes of the first n positive integers: $1^3 + 2^3 + 3^3 + \ldots + n^3$. With some explanation and observations, the following figure of a nested sequence of squares illustrates the method.

1. The sides of the square S are divided into segments of lengths $1, 2, 3, \ldots, n$. Recall Euler's method for summing the first n positive integers. He listed them in a column in ascending order. In a parallel column, he listed the n integers in descending order. He then added the numbers occupying the similar positions in the two columns and summed the results. Because each addition gives $n + 1$ and there are n positions, the resulting sum is $n(n+1)$. Finally since this procedure counts each of the n integers twice, the desired sum is $\frac{n(n+1)}{2}$. That is,

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

Thus square S has side length $\frac{n(n+1)}{2}$.

2. The area of region A_1 is 1 or 1^3.

3. Region A_2 (the "L" shaped region) is composed of three rectangles as shown by the drawing. Thus the area of region A_2 is $2x1 + 2x2 + 1x2 = 8 = 2^3$.

4. Region A_3 (the "L" shaped region) is composed of three rectangles as shown by the drawing. Thus the area of region A_2 is $3x3 + 3x3 + 3x3 = 27 = 3^3$.

5. Draw a diagram to show that the area of region A_4 (the "L" shaped region) is $64 = 4^3$.

6. Draw a diagram to show that the area of region A_5 (the "L" shaped region) is $125 = 5^3$

Is the pattern clear? The area of square S is the square of the length of its side which is

$$[\frac{n(n+1)}{2}]^2 = \frac{n^2(n+1)^2}{4}$$

The area of square S is also the sum of the areas of the regions $A_1, A_2, A_3, \ldots, A_n$. Thus the result

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

Pretty nice reasoning on the part of Abu!

[4] Queries

1. The recent N.C.A.A. basketball playoffs began with 64 teams. How many games were required to produce the national champion? How many games would have been necessary if the tournament had begun with 27 teams?
2. A class of 33 students took a test. What is the largest number of students who could have received a score larger than the class average?
3. Do the graphs of $y = e^x$ and $y = \ln(x)$ intersect? Explain your answer.

[5] Final Exam Questions

1. The equations for the functions f and g, whose graphs are shown in the following plot, form a system of two equations in variables x and y. Circle the solution of that system from the four choices given.

   (a) (-1,-2) (b) (3,-2) (c) (-2,1) (d) 2,1)
2. Consider the system of equations $2x - y = 1$, $x + y = 2$. From the following four plots, encircle the one that represents this system.
3. Determine how far Jason throws the javelin if he lets it fly with a velocity of $v = 60$ ft/sec at a height of 6 feet at an angle of $\frac{\pi}{4}$ with the horizontal. The parametric equations for the position of the javelin are:
   $x(t) = v * cos(\frac{\pi}{4})$
   $y(t) = -16t^2 + v * sin(\frac{\pi}{4}) + 6$
4. Two glasses of 40^0 water are placed on a counter in a 70^0 degree room. One glass contains ice cubes and one does not. On the same axes, sketch the temperature curves of the water in the two glasses. Label the curves and indicate the long term water temperature.

[6] Notices

1. The next issue of the *Vision - Potential* Newsletter will appear in Septemper 2000. The Deadline for contributions to the September Newsletter is Monday, September 4, 2000
2. A dissemination workshop for the Contemporary College Algebra program will be held 4-7 June 2000 at Clark Atlanta University, Atlanta, GA. For application forms and information related to the workshop, contact Dr. Alex Fluellen, Dept. of Mathematics, Clark Atlanta Univ., 223 James Brawley Dr., Atlanta, GA 30314-4391.
   (404) 880-8007 afluelle@cau.edu
3. To subscribe to this Newsletter or to submit articles write to Dr. Della Bell, Chair, Dept.of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

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The pencil with an attached eraser was first patented in 1858.