Vision - Potential

Vision Within Every Instructor — Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

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[1] Change

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V.P. and Global Information Technology Controller

J. P. Morgan and Company

Hey Buddy! Can you spare some change? Change is constant and pervasive. On a daily basis, our world is evolving at an ever-increasing rate, driven largely by technical innovation. The advent of the Internet has provided unparalleled global access to information, and will completely reinvent society. Our success in this brave new world will be determined by our abilities to embrace and drive change.

Growing up, I used to think that the world was a fairly stable place. Technology advancements, to my mind, were the stuff of history books (the cotton gin, the light bulb, vulcanized rubber). My grandparents would tell me about the first cars, and I thought the early 1900's must have been an exciting time to be alive. As it turns out, we live in pretty interesting times, ourselves.

My first computer experiences were a Tandy 60 that Dad brought home. It was a keyboard that you would connect to a (black & white) TV set. You could write and run BASIC programs on it, but that was about it. In high school, my friends and I were using IBM punchcards to write and store programs. Thinking back, these first experiences sound quaint (and quite distant), but the dark ages of computing were only about 20 years ago.

Until recently, technological advances were closely guarded secrets, both by private business and by governments. Do you remember when the Speak-N-Spell toy could not be exported, as its primary microchip was a technical marvel? The globalization of the marketplace has eroded this territorial perspective, and will continue to do so. Increasingly, we are becoming a global melting pot, as the practical hurdles of global industry are reduced. As that happens, technology is brought to the forefront of the consumer mindset. It amazes me that there is a street vendor on Wall Street who sells only programming texts! Amazon.com maintains "reader circles" to track demographics. In Edison, New Jersey, 9 of the top 10 best sellers were pro-

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gramming texts (the 10th was "What to Expect When You're Expecting"). There is a thirst for technical knowledge, as people want to participate in the new world order.

In response to the changing business landscape, private companies are constantly restructuring and reengineering their processes. Creating and maintaining a competitive advantage is a real challenge, as organizations are very quick to mimic successful strategies. As an employee, all of this change translates to personal opportunity. Over the last 10 years, I have worked for 4 multinationals, and had 11 managers. For the last 6 years, my responsibilities changed on a monthly basis. While this may sound chaotic, it has been a challenging and rewarding ride. And the roller coaster is speeding up!

A word of caution: Uncontrolled change is destructive. A former employer used to evaluate its employees' appetite for change, but did not question the underlying purpose. As a result, I saw many different managers driving changes in a haphazard manner, often with cataclysmic results.

So, what does one do in this environment? You have a choice - you can either grab the bull by the horns or you can sit on the sidelines. If you choose to be proactive, you can create many opportunities to influence your work environment. Today's employer looks for the person who can identify business opportunities, generate enthusiasm for their ideas, coordinate a multi-functional project team, and deliver value to the firm. If you opt to be passive, you are voluntarily surrendering yourself to whatever changes happen. It's your choice.

[2] Queries

a. Is it possible to assign positive integers to ten people such that the average of the numbers is twice the median? If so, give an example and determine if there is more than one possible example. If not, explain why such an assignment of numbers is not possible.

b. Explain the existence of the "hole" in the following diagram.

[3] Perfect Squares

A *perfect power* is the nth power of an integer where n is a natural number greater than one. Perfect squares have been part of numerology since at least Pythagorean times and probably before. In 1225, Fibonacci published a treatise on perfect squares, *Liber Quadratorum*, in which he wrote

> I thought about the origin of all square numbers and discovered that they arise out of the increasing sequence of odd numbers; for the unity is a square, namely 1; to this unity is added 3, making the second square, namely 4, with root 2; if the sum is added to the third odd number, namely 5, ...

How do you think Fibonacci realized that the perfect numbers arise out of the increasing sequence of odd numbers? What follows is the beginning of three possible approaches: a graphic, a numeric, and a symbolic approach. Your task is to develop each of the three approaches justifying Fibonacci's insight.

Graphic Represent a positive integer geometrically as a set of unit squares. Thus 2 is represented by two unit squares, 3 is represented by three unit squares, and so on. Now consider the following construction:

Numeric Show that the nth odd positive integer can be expressed as $2(n-1)+1$. For example, the third odd positive integer, 5, can be expressed as $2(3-1)+1$. The fifth odd positive integer, 9, can be expressed as $2(5-1)+1$. Is Fibonacci correct when he says that n square plus the $n+1st$ odd positive integer is equal to $(n+1)$ square? Explain.

Symbolic Show that the nth odd positive integer can be expressed as $2n-1$. For example, the third odd positive integer, 5, can be expressed as $2(3)-1$. The fifth odd positive integer, 9, can be expressed as $2(5)-1$. The sum of the first n odd positive integers can be expressed, using summation notation, as $\sum_{k=1}^{n} 2k-1 = 2\sum_{k=1}^{n} n + \sum_{k=0}^{n} 1$. Find out how to evaluate the two summations and then see if the sum of the first n positive odd integers is equal to n square.

[4] Understanding a Car Advertisement

On March 10, 2000, Mazda advertised a "Progressive Purchase" plan for buying a 2000 MAZDA PROTEGE LX. The ticket price is \$14,940 and the "Progressive Purchase" requires you to pay 10% down, then pay \$144/month for the first six months, \$216/month for the next six months, and finally \$287/month for the next forty-eight months. Is this good deal? Mazda Dealers say it is. In fact, the ad claims that financing the car over sixty months using the "standard" payment of \$287/month results in an Annual Percentage Rate (APR) interest of 10.2%.

For comparison purposes, approximate the APR for the "Progressive Purchase" plan by computing the average monthly payment under the Plan and then using that average payment as the payment for a 60 month loan with equal monthly payment. (You will need to compute the amount financed.)

Hint: Develop a model for the situation (see Problem #1 in Section 3.2 of text *Contemporary College Algebra*, published by McGraw-Hill Co.). Start by defining variables:

$a(n)$ = amount due after the n^{th} payment
$a(0)$ = amount to be financed
r = monthly interest rate = $\frac{APR}{12}$
p = (average) monthly payment
$a(60) = 0$ (loan paid off)

The recursive sequence model format is
(Next Month's Balance) = (Present Balance)
+ (Interest) - (Payment)

Thus $a(n) = a(n-1) + r * a(n-1) - p$.

Iterating, recognizing the geometric series pattern, summing that series, and solving gives

$$a(k) = (1+r)^k * [a(0) - \frac{p}{r}] + \frac{p}{r}$$

Substitute for $a(0)$ and p and solve for r and then for APR.

How does your APR figure compare to a bank's APR figure for a 60 month car loan?

[5] Gross Domestic Product

The nature of the United States economy has changed dramatically during the past century. In the early portion of the century, value was represented by heavy industry - steel mills, railroads, large auto plants, petrochemical complexes. The emphasis appeared to be on building bigger and bigger. The

century ended with ideas replacing physical bulk as creaters of value. Alan Greenspan, Federal Reserve Chairman, recently said "Today, economic value is best symbolized by exceedingly complex, miniaturized integrated circuits and the ideas, the software, that utilize them. Most of what we currently perceive as value and wealth is intellectual and impalpable."

Despite the changes in the fundamental driving forces in our economy, the Gross Domestic Product (GDP), when measured at ten year intervals, appears to have followed a "regular" pattern. The following table shows the U.S. GDP measured in 1996 dollars. (The data was obtained by estimating the amounts from a graph in the Outlook Section of the New York Times, December 20, 1999.) What is the pattern? Form a third column showing the ratios of the GDPs for successive periods. These ratios approximate the growth rate of the GDP.

$Year$	GDP in 1996 dollars (trillions)
1900	0.444
1910	0.500
1920	0.778
1930	1.000
1940	1.111
1950	1.667
1960	2.333
1970	3.556
1980	5.000
1990	6.667
1999	8.889

Form a multiplot of the data, a quadratic regression curve, and an exponential regression curve. Research and then write a one page essay describing which regression curve is the most appropriate given the fact that the data is only taken at ten year intervals and thus may not represent the effects of unusual economic activity such as a depression. A few aspects to consider are: the effects of the major wars - World War I and II, Korean War, Vietnam War on the GDP; the effect of the major economic expansions (1) March 1991 -, (2) February 1961 - December 1969, (3) November 1982 - July 1990, (4) June 1938 - February 1945, (5) March 1975 - January 1980. You might also want to consider if the growth of the GDP is related to the size of the GDP. (Recall that the characteristic property of an exponential function is that its growth rate is proportional to the amount present.)

[6] Notices

1. The Deadline for contributions to the April Newsletter is Monday, April 3, 2000

2. Dr. Della Bell (Texas Southern University, Houston, Texas) recorded comments made at an MAA Open Discussion on College Algebra Reform held during the Joint Mathematics Meetings in Washington, D.C. last January. The program was organized by Don Small (U.S. Military Academy). To obtain copies of the comments, send an email request to Dr. Della Bell (delladbell@netscape.net)

3. Dr. Anton Glaser (Penn State Univ. at Abington, PA) has prepared a summary of the papers presented in the Contributed Paper Session: "Interdisciplinary Applications for College Algebra" that was held January 19-20, 2000 during the Joint Mathematics Meetings in Washington, DC. The summary contains the title and abstract of each paper along with the name and address of the presenter. Copies of the summary can be obtained by sending an email request to Dr. Anton Glaser (axg4@psu.edu).

4. To subscribe to this Newsletter or to submit articles write to Dr. Della Bell, Chair, Dept.of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

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"You can see a lot, just by looking."
Yogi Berra