Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
Number 23, November 1999

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[1] Why Take College Algebra?

Why is College Algebra required by all majors? Why do several states mandate that a student pass a college algebra course (or its equivalent) during their first sixty hours of college? The primary answer is the need to develop students to become competent, confident, and creative problems solvers. (Learning how to analyze data and learning how to plot and interpret graphs are two other major reasons given for requiring college algebra.) The need for problem solving arises in all disciplines and in all walks of life. Furthermore, all disciplines focus on developing students' problem solving skills as they apply to that discipline. What makes college algebra different? Not all majors require chemistry or sociology or economics, yet problem solving is important in each of those disciplines. The answer lies in the universality of mathematics which results from the abstraction process that is basic to mathematics, particularly to mathematical modeling.

Abstraction is the first step in the modeling process, that is identifying variables. "Let $x = ...$" Over the years x has represented thousands if not millions of different things. In one setting, the equation $y = x + 3$ represents the relationship between Mike's age (y) and Jonathan's age (x). In another setting, this same equation could represent the relationship between the distance Kristie (y) hikes in a day and the distance that Levert (x) hikes in a day. Or, the equation could represent the cost (y) of manufacturing x number of toothbrushes. This one equation is the abstraction of thousands of different relationships. The efficiency and power of mathematics lies in the fact that we can analyze this (abstract) equation and then interpret the results in each of the appropriate settings. For example, because the slope is one, we know that Mike and Jonathan will age at the same rate. Or, in the case of manufacturing toothbrushes, we know that because the slope is one there is no economy of scale. It will cost as much to make the tenth toothbrush as the fifth toothbrush.

The modeling process is the basis for solving

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problems in mathematics. This process is illustrated with the following diagram.

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The universality and power of mathematics lies in the abstraction inherent in the modeling process. The analysis of a model in one situation can be transferred to another situation when a similar model is used. The successful problem solver is always alert to interpreting her model in different settings.

The following three articles in this Newsletter indicate the universality of the mathematical modeling approach to problem solving The first article involves an engineering problem – optimizing the area of a field, the second uses a system of equations to model a chemistry problem, and the third involves a graphical model of a sociology problem.

[2] Fencing in a Field

Professor Bill Wulf, President of the National Academy of Engineering, describes engineering as solving problems subject to constraints. In the following problem, the length of the fence is the constraint.

<u>Problem</u>: Shirley has 1,000 feet of fencing to enclose a rectangular shaped pasture. Her question is, What dimensions will give her the largest pasture?

We start the modeling process, as always, by defining the variables. Let

$x =$ the length of the pasture

$y =$the width of the pasture

The model is:

area$= xy$

subject to $2x + 2y = 1,000$

We solve the model by first expressing y in terms of x from the constraint equation. That is $y = 500-x$. Substituting this expression for y into the area equation gives: area $= x(500 - x)$.
We now plot the area equation and using zoom and trace features, determine the x coordinate of the highest point on the graph. We substitute this value for x into the expression for y to obtain the corresponding value for y.
These values of x and y are the width and length of the largest rectangular shaped pasture that can be enclosed with 1,000 feet of fencing.

(Note: This problem illustrates how technology has lowered the access barriers to content. This used to be a standard application of differentiation problem in a Calculus I course. Technology, in the form of a graphing calculator, has made this problem a college algebra problem.)

[3] Balancing Chemical Equations (Stoichiometry)

Conservation laws are fundamental to science. The law of conservation of mass states that there is no detectable change in mass during the course of an ordinary (non-nuclear) chemical reaction. This means that there must be as many atoms of each element, combined or uncombined, after a chemical reaction as before the reaction. (This law was first stated by Antoine Lavoisier in his work *Traite Elementaire de Chemie* in 1789.) This law is illustrated by the following problem.

Everyone has seen rust – a rusty nail, rust on a car fender, rusty shovel, etc. Rust results from combining iron and water. The chemical reaction is iron plus water produces iron oxide (rust) plus hydrogen gas. The chemists expresses this reaction by writing

$$Fe + H_2 \longrightarrow Fe_4 + H_2$$

<u>Problem</u>: Balance this chemical reaction. That is, determine the number of atoms of each chemical that will satisfy the law of conservation of mass.

We begin the modeling process, as always, by defining our variables. Let

$$\begin{aligned} x_1 &= \text{number of atoms of Fe (iron)} \\ x_2 &= \text{number of atoms of } H_20 \text{ (water)} \\ x_3 &= \text{number of atoms of } Fe_30_4 \text{ (iron oxide)} \\ x_4 &= \text{number of atoms of } H_2 \text{ (hydrogen)} \end{aligned}$$

Our objective is to determine values for these four variables such that

$$x_1Fe + x_2H_20 = x_3Fe_30_4 + x_4H_2$$

Applying the law of conservation of mass to iron (Fe) yields:
$x_1 = 3x_3$
Applying the law of conservation of mass to hydrogen (H) yields:
$2x_2 = 2x_4$
Applying the law of conservation of mass to oxygen (0) yields:
$x_2 = 4x_3$

In order to balance the chemical reaction we need to find values for $x_1, x_2, x_3,$ and x_4 that satisfy the preceding three equations. Becasue we have three equations in four unknowns, we solve for three of the unknowns in terms of the fourth, say x_4. We therefore express the preceding three equations as the following system of equations. This is our model.

$$\begin{array}{ccccc} x_1 & & -3x_3 & = & 0 \\ & 2x_2 & & = & 2x_4 \\ & x_2 & -4x_3 & = & 0 \end{array}$$

The mathematical solution of this system of equations is

$$x_1 = \frac{3}{4}x_4, \quad x_2 = x_4, \quad x_3 = \frac{1}{4}x_4$$

Because the number of atoms must be an integer, we choose x_4 so that all the variables are positive integers. Choosing $x_4 = 4$ means that $x_1 = 3, x_2 = 4,$ and $x_3 = 1$. The balanced equation is then

$$3Fe + 4H_20 \longrightarrow 1Fe_30_4 + 4H_2$$

Balance the following chemical reactions:

a. Ethone plus Oxygen produces Carbon Dixoide plus Water:

$$C_2H_6 + 0_2 \longrightarrow C0_2 + H_20$$

b. Chlorine plus Potassium Hydroxide produces Potassium Chloride plus Potassium Chlorate plus Water:

$$Cl_2 + KOH \longrightarrow KCl + KC0_3 + H_20$$

[4] Small Group In-Class Activity: Were You Ever as Heavy as Tall?

Was there ever a time when your height, measured in inches, was equal to your weight measured in pounds? If so, was there more than one time when the numerical measurements of your weight and height were the same?

The following, outlines steps in formulating a model of the situation. Remember to identify the variables.

1. Assume that a person's weight changes in a linear manner. Develop a linear weight function for the combined members of the group. Let time be the independent variable.

   a. Determine the average birth weight, in pounds, of the members of the group.

   b. Determine the average age of the members of the group and the average weight, in pounds, for the members of the group.

   c. Determine a linear model (a straight line) using the data from parts a. and b.

2. Assume that a person's height changes in a linear manner. Develop a linear height function for the combined members of the group. Let time be the independent variable.

   a. Determine the average birth height (or length), in inches, of the members of the group.

   b. Determine the average age of the members of the group and the average height, in inches, for the members of the group.

   c. Determine a linear model (a straight line) using the data from parts a. and b.

Plot the two lines determined in the preceding two parts. Let time be measured on the horizontal axis and let the numerical measures of weight and height be measured on the vertical axis. This plot is the model.

Do the two lines intersect? If so, determine the point of intersection. Interpret what this point represents in terms of the original problem. What does it mean if the two lines do not intersect? Explain.

A person makes assumptions in order to develop a mathematical model of a real life situation. Having developed a model, one improves the accuracy of the model by relaxing one or more of the assumptions. In modeling the above weight-height scenario, we assumed that both a person's weight and height changed in a linear fashion. Is this a realistic reflection of the real situation? Explain. Improve your model by removing the linear assumptions. Estimate "reasonable" weight and height curves, fit functions to your estimated curves, and then determine the point or points of intersection. Compare these results to your previous results.

Discuss the conditions under which you would be satisfied with your first model and the conditions under which you would insist on your second model. How could you improve on your second model?

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Notices

1. The AMATYC 25^{th} Annual Meeting will be held in Pittsburgh, PA, November 18-21, 1999. Jackie Giles and Don Small will present a four hour workshop: "Contemporary College Algebra: A Reformed Program."

2. The next issue of *Vision-Potential* Newsletter will appear in January. The <u>Deadline</u> for contributions to the January Newsletter is

   <u>Monday, January 10, 2000</u>

   Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are all welcomed. Please send material to Dr. Della Bell, Chair, Dept.of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

3. The Contemporary College Algebra "Team" will be strongly represented at the National Mathematics Meetings in January in Washington, DC.

   a. Della Bell, Ahmad Kamalvand, and Don Small will host a six hour contributed paper session: "Interdisciplinary Applications in College Algebra." (Wed. 2:15-6:00 PM; Thur. 1:00-3:45 PM)

   b. Laurette Foster will be a presenter in Minicourse 10: "Interdisciplinary Lively Applications Projects." (Thur. 8:00-10:00 AM; Sat. 9:00-10:00 PM)

   c. Laurette Foster will be a panelist on the panel "Changing the Academic Culture." (Fri. 1:00-2:20 PM)

   d. Our team will host an "MAA Open Discussion on College Algebra Reform" (Sat. 1:00-3:00 PM)

4. To subscribe to this Newsletter, send your name and address to Dr. Della Bell, Department of Mathematics, Texas Southern University, 3100 Cleburne St.,Houston, TX. 77004

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"A pessimist sees the difficulty in every opportunity, the optimist sees the opportunity in every difficulty." Sir Winston Churchill (1874-1965)