Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
Number 20, April 1999

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[1] Shall I take the Grandmother Clock? or Is More Really Less?

Rev. David Bidnell and his wife Fiona are about to complete their five year appointment in Haiti and return to their home in London. Fiona is an electrical engineer. Both she and David have boxes of books that they would like to take home with them along with their clothes and several Haitian artifacts including a favorite Grandmother Clock. They are concerned about the cost of transporting their belongings, for books in particular are very heavy. As there is no surface shipping available, they must transport by air. Fiona called American Airlines, which flies from Port-au-Prince, Haiti to Miami, Florida and then on to London, England, to ask about charges for excess baggage. The agent in Port-au-Prince gave her the following rates

a. $191.13 for 70 pounds

b. $344.19 for 200 pounds.

c. $1.62 per kg. for 200 kg.

The agent could not explain why some rates were based on pounds and some on kilograms.

Fiona estimates that they have 300 pounds of excess baggage that they definitely need to take with them. However, there is an additional 200 pounds of baggage, including their Grandmother Clock, that they would like to take. This "additional" amount could easily expand to 300 pounds to include a favorite chair.

Task #1: Analyze American Airlines rate structure and determine the costs for the Bidnells' to take 300 pounds and compare it to the cost of their taking 500 pounds. Assume that the data the agent gave Fiona was just three points on American Airlines' rate curve for excess baggage.

Hint: Begin your analysis by:

* Supported by the EXXON Education Foundation and the U.S. Military Academy.

a. Determine the dollars per pound charge for the given data.

b. Plot the converted data measuring rate (in "dollars/pound') on the vertical axis and the pounds on the horizontal axis.

c. Graphically determine a rate function ($r(x)$) by approximating values for the parameters a, b, c such that the graph of $r(x) = ae^{-\sqrt{bx}}+c$ "fits" the three data points.

Task #2: Recognize that the cost ($c(x)$) of transporting x pounds is equal to the product of the number of pounds (x) and the rate ($r(x)$). Therefore, the cost function for transporting x pounds of goods from Port-au-Prince, Haiti to London, England is $c(x) = xr(x)$. Plot the cost function over the interval $[0, 1000]$.

Task #3: Interpret the graph of the cost function. In particular,

a. Are the results from your Task #1 consistent with the graph of your cost function?

b. Can you lower your cost by transporting more weight, say by including boxes of rocks in your luggage?

c. What amount of weight produces the maximum cost? What amount of weight produces the minimum cost? (Hint: Plot the cost function over the interval [0,5000].)

Task #4: If you were a shareholder in American Airlines, how would you feel about its rate structure for excess baggage? Explain.

[2] Linda's Fruit Punch

Linda and her son prepare a large bowl of fruit punch for her son's birthday party. Their special recipe that makes 2.5 gallons of punch is:

Stir 8 cups of sugar and 2 cups of strawberry jam into 1.5 gallons of water
Add: 3 cups grape juice, 3 cups pineapple juice, 8 sliced bananas
Just before serving add: 2 quarts of cherry Koolaid frozen into cubes
and 6 ounces of frozen grapefruit juice concentrate.

The party is about to begin. As Linda goes to the front door to welcome the guests she hears a cry of anguish from the kitchen. "Oh No! Mom, I put the 12 ounce can of frozen grapefruit concentrate in the punch instead of the six ounce can." What to do? The punch bowl is too full to add any water and there isn't time to pour out some of the punch and then dilute the remainder. (Besides, how much punch would have to be removed and replaced with water?)

Having studied Contemporary College Algebra, Linda immediately views the problem in terms of a discrete dynamical system. Hoping some of the guests like a tart punch, she decides to serve the punch and after each cup add a cup of water to the punch bowl. The question she asks her son is "How many cups of punch need to be served before the grapefruit concentration (oz/cup) agrees with the recipe concentration (that is, 6/40)?"

Hint: Let $c(n)$ denote the grapefruit concentration just after the n^{th} serving has been made. Thus $c(0) = 12/40$. (There are 4 cups in a quart and 4 quarts in a gallon.)

Recall that the amount of grapefruit concentrate is equal to the volume of punch times the concentration. Thus $39c(n)$ is the amount of grapefruit concentrate just after the n^{th} cup has been removed and before the cup of water is added. Therefore

$$c(n+1) = \frac{39c(n)}{40} = \frac{39}{40}c(n)$$

The next step is to solve this discrete dynamical system (that is, recursive sequence). To do this, iterate the dynamical system and look for a pattern.

For example:

$c(1) = \frac{39}{40}c(0)$
$c(2) = \frac{39}{40}c(1) = (\frac{39}{40})\frac{39}{40}c(0) = (\frac{39}{40})^2c(0)$
$c(3) =?$
$c(4) =?$
$c(5) =?$

What is the pattern? That is, what is the expression for $c(k)$ in terms of $c(0)$?
The final step is to determine the smallest value of k such that $c(k) \leq 6/40$. Recall that k represents the number of cups.

[3] In-Class Activity: Too Much Hot Sauce

Linda is mixing up two gallons of Shrimp Creole in preparation for a dinner party. Unfortunately Linda misreads the recipe and adds 2 tablespoons of Hot Cajun Sauce per gallon rather than the 2 teaspoons per gallon called for in the recipe. (Note that there are 3 teaspoons in a tablespoon. Thus she has added 12 teaspoons of hot cajun sauce instead of 4 teaspoons!) What to do? She certainly doesn't want to throw out the two gallons of shrimp creole and besides some people might like HOT shrimp creole. She decides to serve the shrimp creole over rice, two cups to a serving. After each serving, she adds two cups of a white sauce to the shrimp creole bowl. After how many servings of shrimp creole will the concentration of Hot Cajun Sauce in the shrimp creole be reduced to that specified in the recipe?

Hint: Work Linda's Fruit Punch problem.

[4] In Class Activity: Is Kepler Correct?

In 1619 Johann Kepler (1571-1630), a German astronomer and mathematician, formulated his law of planetary motion relating the period of revolution of a planet to its average distance from the Sun. The following table gives the average distances d of the planets from the Sun (taking the unit of measurement to be the distance from the Earth to the Sun) and their periods T (time of revolution in years).

Planet	d	T
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.861
Saturn	9.541	29.457
Uranus	19.190	84.008
Neptune	30.086	164.784
Pluto	39.507	248.350

a. Plot the data (T on the vertical axis and d on the horizontal axis).

b. Graphically fit a curve to the data points. Hint: Note that the graph of $y = x$ lies under the data points and the graph of $y = x^2$ lies over the data points. This suggests experimenting with graphs of $y = x^k$ for $1 < k < 2$.

c. Kepler's Third Law of Planetary Motion states that "The square of the period of revolution of a planet is proportional to the cube of its average distance from the Sun." Does your result in part b. agree or disagree with Kepler's result? Explain.

This problem was adapted from exercise 6, p. 83 in James Stewart's Calculus, Concepts and Contexts Brooks/Cole Publishing Co.)

[5] Writing Assignment: Catfish Farming

The following request was received from Mr. Hook of Catfish Farms, East Bait, Texas.

I am considering developing a commercial catfish farm on a piece of land that I own that has a reasonable access to a large water supply. An experienced catfish farmer has estimated the following expenses for me in terms of x schools of fish. Each school contains 100 fish.

a. $\$x^2$ = cost of piping and filtering water.

b. $\$15x$ = cost of stocking and feeding x schools of fish to mature size.

c. \$3,000 = cost of building a fish pond (excavating, banking, piping, and so on).

d. \$4 = retail price for a mature catfish

I have a total budget of \$3,250 for the project. I must realize at least a 20% profit in order to risk this investment.

Please analyze this situation and advise me on the number of fish that can be raised within my financial constraints and the percent of profit I can expect to receive. Thank you very much for this assistance.
Sincerely yours,
Bent Hook

P.S. Please stop by, as my guest, for our Saturday night fish fry.

Your task is to write a letter to Mr. Hook answering his questions and explaining how you obtained your answers. Apparently Mr. Hook is better at fishing than he is at mathematics. Thus the descriptions of your analysis must be very complete and clearly stated.

[6] Queries

a. If the average test score in your College Algebra class is 78 and the median grade is 60, what can you say about the distribution of test scores?

b. How many minutes will there have been from 0 AD to 2000 AD? (Assume that every fourth year is a leap year.) How does this number compare to the size of our National debt? How does the number of minutes compare to the value of the Exxon-Mobil merger that is estimated to be 80 billion?

[7] Notices

1. The Mathematical Sciences and Their Applications Throughout the Curriculum (MATC) will hold a workshop this summer at Indiana University (Bloomington) July 9-10, 1999. Laurette Foster will take part in presenting a minicourse on "Reforming College Algebra."

2. The Fourth Annual Retreat of the HBCU College Algebra Reform Consortium will be held September 30-October 2, 1999 at Wiley College, Marshall, Texas.

3. The AMATYC 25^{th} Annual Meeting will be held in Pittsburgh, PA, November 18-21, 1999. Jackie Giles and Don Small will present a 4 hr. workshop: "Contemporary College Algebra: A Reformed Program."

4. The Joint Mathematical Meetings will be held in Washington, D.C. next January. Della Bell and Ahmad Kamalvand will host a Contributed Paper Session on "Reforming College Algebra."

5. The next issue of *Vision-Potential* Newsletter will appear in September. The Deadline for contributions to the September Newsletter is

   Monday, September 6, 1999

   Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are all welcomed. Please send material to Dr. Della Bell, Chair, Dept.of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

6. To subscribe to this Newsletter, send your name and address to Dr. Della Bell, Department of Mathematics, Texas Southern University, 3100 Cleburne St.,Houston, TX. 77004