Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*

Number 19, March 1999

Contents

---

[1] Projectile Motion from Aristotle to Galileo

Dick Jardine
U.S. Military Academy
West Point, NY

It's Spring time, the time of year when baseballs arc across the Florida sky, launched from the bats of major league sluggers. And what curve best describes the motion of a projectile, such as a baseball? The correct answer to that question seems to have evaded some of the most capable minds of Antiquity, and it was not until Galileo that the issue was resolved.

Aristotle (384-322 B.C.) held the belief that when an object like an arrow was shot into the air, it was propelled by "antiperistasis." As the arrow moved through the air, an empty space was created behind it. Since nature abhors a vacuum, air rushed rapidly into the empty space behind the arrow and propelled the arrow forward in a straight line. This propulsion continued to push the arrow along a linear path until the arrow stopped, at which time the arrow plummeted straight down to the ground.

The Aristotelians believed that all terrestrial motion was linear, with antiperistasis as the driving force. Curved paths were reserved for the motion of the sun, moon, and other celestial bodies. Educated people accepted the teachings of Aristotle almost without question until the Renaissance. So the predominant view for over a thousand years was that projectiles moved in linear paths in the direction in which they were launched, until they ran out of impetus, and then fell to the ground. Evidence of this notion can be seen in illustrations contained in books such as Daniel Sentbach's *Problematum astronomicorum et geometricum sectiones septum* published in Switzerland in 1561, in which there are drawings of cannon trajectories that are straight lines.

Sentbach published his work apparently unaware of the work of the mathematician Niccolo of Brescia. Niccolo is also known as Tartaglia ("the stammerer") because of a speech impediment caused by

* Supported by the EXXON Education Foundation and the U.S. Military Academy.

wounds he sustained when his home city of Brescia was sacked by the French in 1499. Despite his speech difficulties and other hindrances, Niccolo taught himself mathematics, so well that he became one of the leading mathematicians in Europe.

Recognized as one of the best mathematical minds, he was consulted on the maximum range of the dominant weapon of the time, the canon. Tartaglia published two works, *Nova scientia* in 1537 and *Quesiti et inventioni diverse* (1546) which contained significant information about projectile motion.

Tartaglia described a projectile's path as having three parts. The first part was a straight path, caused by the force of gunpowder on the cannonball. The second was a circular arc caused by gravity, and the third was another line as the cannonball fell straight to the ground in a perpendicular path.

Tartaglia's notions about projectile motion were the accepted "facts" until Galileo (1564-1642) proved otherwise. Galileo was familiar with the work of the Greek mathematician Apollonius (250-175 B.C.) on conic sections, which includes descriptions of curves such as ellipses, parabolas, and hyperbolas. Galileo experimented with a ball rolling off the edge of a table, and noticed that the path of the ball was precisely that described by Apollonius as a parabola. In 1608, Galileo proved, using the geometric arguments appropriate in his day, that the path of a projectile was indeed a parabola.

Certainly air resistance, wind, and the flight dynamics of the ball alter the path from that of the true parabola described by Apollonius and Galileo. But when you see Sammy Sosa launch a baseball out of Wrigley Field, be confident that the curve the ball follows is a parabolic trajectory, and be glad that Galileo already proved that result, so that you can sit back and enjoy the game.

[2] Estimating How Many Board Feet are in a Log

When a logger buys stumpage, he pays the landowner for the right to cut the timber on her land. The amount paid depends on the estimated value of the lumber sawed from the trees the logger cuts. The value of lumber is measured in board feet. (A board one foot long, one foot wide, and one inch thick measures one board foot.) Thus a logger needs to know how to estimate the number of board feet in a standing tree in order to determine a reasonable stumpage fee. How is that done? In the following data table, g is the girth (circumference) of a tree measured in inches six feet off the ground and b is the corresponding number of board feet that can be cut from the tree.

g	17	19	20	23	25	28	32	38	39	41
b	19	25	32	57	71	113	123	252	259	294

Plot the data with the vertical axis representing the number of board feet and the horizontal axis representing the girth. Do any of the data points appear to be wrong? If so, which ones? Give reasons for your answer.

Do the following

a. Graphically fit a straight line to the data. Is the equation of your straight line a good model for estimating the number of board feet in a tree? Explain your reasoning? How important is it that the graph of a reasonable model passes through the origin?

b. Graphically fit a quadratic curve to the data. Is the equation of your quadratic function a good model for estimating the number of board feet in a tree? Explain your reasoning?

c. Graphically fit a cubic curve to the data. Is the equation of your cubic function a good model for estimating the number of board feet in a tree? Explain your reasoning?

Which one of your three functions (linear, quadratic, cubic) provides the best model based on the given

data? Why? With respect to your best model, do any of the data points appear to be wrong? If so, are they the same points that you identified earlier?

[3] Logarithms for Scaling Large Numbers

A healthy human being can distinguish different levels of sound over a tremendous range of intensity values. The loudest sound a person can hear without ear damage has an intensity that is approximately one trillion (1,000,000,000,000) times the intensity of the softest sound a person can hear. How can you scale sound over such a range in a way that could be easily understood by the general public? For example, what does a manufacturer of a "boom box" print on the label to describe the range of sounds produced by the boom box? Or, how do legislatures rate the noise of jet aircraft taking off and landing at an airport? Or, how noisy is a quiet library?

These questions and similar ones are answered by first considering the ratios (percentages) of the intensities of various activities to the intensity of the least audible sound which has been standardized to be $I_0 = 10^{-12}$ watts per square meter (W/m^2). For example, the intensity of a soft whisper is listed as $I = 10^{-9}$. Thus the ratio is $I/I_0 = 10^{-9}/10^{-12} = 10^3$. A second consideration in forming a usable scale to relate large numbers is to realize that the logarithm of a number grows much more slowly than the number. The following table shows the intensity of various activities, the ratio of intensities, and the common logarithm of the ratio.

Sound Source	*Intensity* (W/m^2)	*Ratio* (I/I_0)	$log(I/I_0)$ Breathing
Breathing	10^{-11}	10^1	1
Library	10^{-8}	10^2	2
Conversation	10^{-6}	10^6	6
Busy Traffic	10^{-5}	10^7	7
Niagara Falls	10^{-3}	10^9	9
"Rock" Concert	10^0	10^{12}	12
Jet Takeoff	10^3	10^{15}	15

The common scale that is used to measure noise is called the decibel (dB) scale (named after Alexander Graham Bell, 1847-1922). This scale multiplies the common log values by 10 to spread out the values in order to distinguish between more sound sources. The decibel scale function is defined by

$$dB = 10log(I/I_0)$$

Answer the following questions

a. Why is the common logarithm used (base 10) in the decibel scale rather than the natural logarithm (base e)?

b. What is the decibel rating of a running Jackhammer that has an intensity rating of 3.2x10 W/m^2?

c. If the intensity of a barking dog is $3x10^6$ W/m^2, what is the decibel rating for two barking dogs?

d. How many barking dogs would be needed to produce a decibel rating that is three times the decibel rating of one barking dog?

[4] In-Class Activity

A group of ladies decided to form a quilting group. For their first meeting, they agreed to bring scraps of cloth from which they could cut rectangular pieces. They would then sew these rectangular pieces together to make a quilt. Pearl, a dressmaker, brought several pieces that were shaped like the following

Pearl presented the following two challenges to her friends (and to you)

a. What is the maximum area that can be enclosed by a rectangle that has one vertex at the origin and its diagonal vertex on the curved edge?

b What is the maximum perimeter for a rectangle that has one vertex at the origin and its diagonal vertex on the curved edge?

[5] 10 Minute Writing Assignment

Write an essay (no more than one page) discussing inverse functions. Your essay should include

a. What the term "inverse function" means.

b. The condition(s) that a function must meet in order to have an inverse function.

c. Examples of real life functions that have inverses and examples of real life functions that do not have inverses.

d. How you can tell from the graph of a function whether or not it has an inverse function.

[6] Queries

a. (This question was taken from the September/October 1998 issue of Quantum, published by the National Science Teachers Association in cooperation with Springer-Verlag New York, Inc. B241, p.19)

Sam says there are three natural numbers x, y, z that satisfy the equation 28 x + 30 y + 31 z =365. Is he right?

b. The March seventh New York Times announced that the United Nations had decided to use its peace prize money to build a monument commemorating all peacekeepers who lose their lives on duty. On December 10, 1988 a Nobel Peace Prize of $339,000 was awarded to the United Nations for its peacekeeping efforts around the world. The United Nations deposited the money in a bank while it debated how it should be spent. Eleven years later the decision has been made and the value of the original prize has increased to $700,000. What annual rate of interest over the eleven years would give the present value of the prize? What interest rate compounded monthly would give the present value of the prize?

[7] Notices

1. The Fourth Annual Retreat of the HBCU College Algebra Reform Consortium will be held September 30-October 2, 1999 at Wiley College, Marshall, Texas.

2. The Deadline for contributions to our April Newsletter is

   Monday, April 5, 1999

   Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are all welcomed. Please send material to Dr. Della Bell, Chair, Dept.of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.

3. To subscribe to this Newsletter, send your name and address to Dr. Della Bell, Department of Mathematics, Texas Southern University, 3100 Cleburne St.,Houston, TX. 77004

- -

"Hold yourself to a higher standard than anyone else expects of you."

Henry Ward Beecher