Vision - Potential

Vision Within Every Instructor – Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
Number 10, January 1998

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[1] Our Students' Futures

At last June's Mathematics Across the Curriculum Workshop held at Villanova University, Linda Banecker of Lockheed-Martin Data Systems said "(Because) The technology and markets change so quickly, we assume we need to teach employees about our business. What they need to know coming in, is **how to communicate**, **how to work effectively in groups**, and **how to learn**." (Note the use of technology is involved in both the communication and how to learn categories.)

There are two big difficulties to overcome before we can seriously focus on developing communication skills. They both involve time. They are

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a. Providing class time, and

b. Providing grading time.

Student presentations take class time and the loss of class time usually translates into cutting down on content, a difficult thing for most instructors to accept. Writing assignments need to be graded and their grading usually takes longer than grading a set of mechanical type problems. Helping our students develop employable skills as well as to become contributing members of society requires that we make the time available to focus on developing communication skills. A large and growing number of instructors are stressing the importance of the weekly ten minute, in-class, writing assignment.(For example, write a brief essay on how to add fractions.)

Group work provides students with good experiences for entering the work force as well as enriching and expanding their learning. The synergism in group work is a powerful force that can greatly expand students' thinking. Group work also contributes to the development of communication skills because members must debate with each other in order to resolve the problem and ensure everyone's understanding of the solution. Group work may be the most effective way of engaging students in the material.

A major ingredient in developing students to learn how to learn is to create and strongly support a sense of inquisitiveness and fun in working problems. Thus we need to offer students opportunities to work in small groups on interesting problems that will challenge their ingenuity and provide them with discovery type experiences.

We should encourage students in their groups (as well as individually) to "What If" exercises in order to gain more conceptual understanding of the problem involved and also to help them open up new situations. Looking on an exercise as a special case of a larger problem begs the questions

a. What is the larger problem?

b. What is a solution method for the larger problem that can be specialized to the given exercise?

When students are asking these questions, learning is taking place,

The small group activities presented in this Newsletter (as well as in previous and future Newsletters) are intended to offer such opportunities.

[2] Class Activity: 1998 U.S. Population

This is a 10-15 minute small group, in class activity. Each group should have at least one calculator. At the end of the time period, the instructor calls on different groups to present their work and explain their reasoning. At the end of the session, the instructor can invite students to bring to class other problems based on newspaper articles.

On December 28, 1997 a New York Times article stated that the Census Bureau projected the U.S. population would be 268,921,733 on January 1, 1998. This would represent a nine tenths of one per cent increase over the January 1, 1997 population. What was the population on January 1, 1997?

The article stated that the population on January 1, 1990 was 248,765,170. Is the nine tenths of one percent gain in population during 1997 more or less than the average one year percentage gain since 1990? Explain your reasoning.

The Census Bureau expects the population to grow by 2.3 million persons in 1998. Would the percentage rate of this projected growth be more or less than the yearly average percentage rate from 1990 through 1997. Explain your reasoning.

The projected number of births during 1997 was 3.9 million compared to the 4.2 million births in 1990. Compute the average yearly decline in the number of births from 1990 through 1997.

[3] Small Group Activity:Recursive Sequences

A recursive sequence is a listing of numbers such that each number, after the initial numbers, is determined by one or more of the preceding numbers. In this activity, we will define an "addition sequence" as a recursive sequence with the properties that

a. The first two numbers are initial numbers, and

b. Each succeeding number is the sum of the two preceding numbers.

For example,

2	3	5	8

, is an addition sequence. The first two numbers, 2 and 3, are the initial numbers, the third number 5 is the sum of 2 and 3, and the fourth number 8 is the sum of 3 and 5.

If the above sequence was extended to six numbers, how would you determine the fifth and sixth numbers?

2	3	5	8		

A more interesting problem is to complete an addition sequence when only two numbers are given, say the first and last numbers. For example, fill in the missing numbers in

1			9

There are 5 questions to answer.

1. Develop an algorithm (i.e., method) for filling in the missing elements in the following addition sequences.

Hint: After you have successfully completed three of the addition sequences, stop and reflect on how you completed the sequences. Describe your method (i.e., your algorithm) to someone else and then test it on another sequence. If you can't develop an algorithm, work another addition sequence and then reflect again. Keep doing this until you can formulate an algorithm. (Make up additional addition sequences, if needed. That is, write down four cells and put integers in the first and fourth cells.)

a.

1			5

b.

1			3

c.

2			4

d.

3			5

e.

2			6

f.

3			7

g.

4			6

2. Test your algorithm from question 1. by applying it to:

1					13

and

2					11

3. Modify your algorithm to complete the addition sequence

	1		3		

4. Repeat questions 1. and 2. replacing the addition operation with subtraction.

5. Repeat questions 1. and 2. replacing the addition operation with multiplication.

[4] The Power of the Inverse

The following is a somewhat fanciful story of the development of the real number system whose purpose is to emphasize the importance of always considering the inverse when developing an operation.

"In the beginning" (well within a few thousand or million years of the beginning) people began counting on their fingers and gradually developed the natural numbers (set of positive integers), which we denote by **N**. The first arithmetic operation, addition, was probably developed to simplify counting. Because early mathematicians were as prone to make mistakes as present day ones, there was a need to develop a method to undo the operation of addition (i.e., determine the inverse of addition). Thus subtraction was defined as the inverse to addition. Although it took a surprisingly long time before zero was introduced into the number system (see the November '97 Newsletter), that eventually happened. We now had a new number system, the system of integers denoted by **I**. The need for an inverse of the addition operation led to a generalization of the natural numbers to the set of integers.

Men (and women) are impatient and they felt a need to become more efficient in their counting. So the second arithmetic operation, multiplication, was developed. (Multiplying seven times nine is more efficient than adding nine to itself seven times, especially when using fingers as the counting tools.) The existence of two operations increased the potential for mistakes and thus there was a need to develop a method to undo the operation of multiplication (i.e., determine the inverse of multiplication). The inverse, division, led to a new number system called the rational numbers which are denoted by **Q**. This system contained the system of integers. Once again the need for an inverse led to a generalization, this time from the integers to the rational numbers.

Men and women are seldom satisfied and so they demanded even more efficient methods for computing.The next step was to develop the power function concept (raising a number to an integer power). For example, it is more efficient to work with 2^4 rather than multiplying 2 by itself 4 times which is better than adding 2 to itself 8 times. However nothing comes without a cost, and including another operation increased the potential for mistakes. Thus there was a need to develop a method to undo the power function (i.e., determine an inverse). Although it was easy to square a number, it was not easy to undo the squaring operation in the rational number system. In fact, it was impossible since the square root of two is not a rational number. Hence it was necessary to generalize the rational number system to include radicals, the inverses of the power functions. Thus once again developing an inverse led to a generalization of the number system.

Men and women insisted on even more efficient computing techniques especially for the astronomers. John Napier (1550-1617) answered the call by developing logarithms, an operation that mathematical historian Howard Eves calls, one of the greatest labor saving devices in the history of computing. As always with a new operation there was a need to develop its inverse, which in this case is the exponential function. This allowed for raising numbers to irrational as well as rational powers, which in turn led to generalizing the rational number system to the real number system, denoted by **R**. So once again developing an inverse led to a generalization of the number system.

The moral of this story is that when learning a new operation, one should also learn the inverse operation for that may lead to an interesting generalization.

[5] Writing Assignment: What is a Mathematical Pronoun?

Mathematics is the language of science. As such, it must have objects that corresponds to pronouns. What is a mathematical pronoun?

Write an article supporting or rejecting the statement: " A mathematical variable is a mathematical pronoun."

Your article should include a dictionary definition of "pronoun" and a problem or paragraph from your math text that involves a variable. Explain how the use of the variable in the piece from your math text agrees or does not agree with the dictionary description of a pronoun.

[6] Queries

a. How Many Pigs? A quick count of the animals on Jones' Farm revealed that there were 70 animals, some were pigs and some were chickens. If the combined number of legs was 200, how many pigs were there? Is there only one correct answer?

b. The number zero was not in anyone's number system when Christ was born(See the November '97 Newsletter). Thus the first day under the Julian calendar was January 1, 1. That is January first of year one, not January first of year zero. Given that a millennium is one thousand years, when will we (or should we) celebrate the new millennium?

c. Is $\sqrt{1996} - \sqrt{1995} > \sqrt{1998} - \sqrt{1997}$? Explain your reasoning.
Is $\sqrt{x+1} - \sqrt{x} > \sqrt{x+3} - \sqrt{x+2}$ for all positive x? Explain your reasoning.
Hint: Think about the shape of the graph of $f(x) = \sqrt{x}$.

[7] Notices

The Deadline for contributions to our January Newsletter is

Friday, January 16, 1998.

Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, CBL activities, announcements, and so on are all welcomed. Please send material to Dr. Della Bell, Chair, Dept. of Mathematics, Texas Southern University, 3100 Cleburne St., Houston, TX 77004.