Vision - Potential

Vision Within Every Instructor - Potential Within Every Student

Newsletter of the HBCU College Algebra Reform Consortium*
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[1] Why Conceptual Understanding is Important in College Algebra

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Several years ago, our department conducted an experiment in which two sections of our college algebra/trigonometry course were taught stressing conceptual understanding/modeling while the other two were taught in the traditional algebraic drill-and–skill format with graphing calculators required for both. Although placement was random, the students in the modeling sections had considerably lower scores on our placement test than those in traditional sections, so the "traditional" students actually had better algebraic skills to start with.

Our study compared student performance on 10 common questions on the final exams. Because some faculty felt it was unfair to give the traditional students problems emphasizing conceptual understanding or realistic applications, these questions were essentially algebraic, although this wasn't a major focus of the modeling sections.

Surprisingly, the modeling students outperformed the traditional students on 7 of these 10 questions. The results on one question are particularly telling. The students were given enrollment information on a college two years apart and were asked, among other things, to write a linear function giving enrollment as a function of the year, several predictive questions, and to explain, using an English sentence, the meaning of the slope. Almost every modeling student gave a response demonstrating an understanding of slope. Some typical responses (including several erroneous answers) are:

Every year, enrollment increases by 78 students.
The yearly increase of enrollment is 78 per year.
The enrollment increases by 78 students every year.
Every t year it will increase by that #.
The slope is the growth in enrollment.
The enrollment of students goes by 2/15 x a year
The slope is the amount of students that enroll per year.
Every year student population increases by 78 students.
The population of students increases by 78 every 2 years.
The enrollment increases 78 students

* Supported by the U.S. Military Academy.

every one year.

The yearly increasing in students enrollment is 78.

It means that the number of students enrolling per year is 78.

Yearly increases in students enrollment is 78 every year.

Every year there will be an increase of 78 students.

Slope is the increase in the # of enrolled students per year at Brookville college.

This means that for every year the number of students increases by 78.

The slope means that for every additional year the number of students increases by 78.

For every year that passes, the student number enrolled increases 78 on the previous year.

As each year goes by, the # of enrolled students goes up by 78.

This means that every year the number of enrolled students goes up by 78 students.

Number of students enrolled increases by 78 each year.

Student enrollment increases by an average of 78 per year.

For every year that goes by, enrollment raises by 78 students.

That means every year the # of students enrolled increases by 2,780 students.

For every year that passes there will be 78 more students enrolled at Brookville college.

The slope means that every year, the enrollment of students increases by 78 people.

Brookville college enrolled students increasing by 0.06127.

Every two years that passes the number of students which is increasing the enrollment into Brookville College is 156.

This means that the college will enroll .0128 more students each year.

In contrast, only about one-third of the traditional students could provide a meaningful answer. Another third either skipped the question or rephrased the formula for ■y/■x into words. Some typical responses are

> The slope indicates the average increase per yr.
>
> The difference in (y2 – y1)/ (x2 – x1)
>
> Every year there is an increase of 78 students.
>
> Slope would be a constant increase or decreasing of a line. So if you would enroll 1 person a year the constant would be one and so would the slope.
>
> Enrollments per year.
>
> The slope is the amount of the new students the school gets each year.
>
> The point in which the # of students is increasing.
>
> The meaning of the slope is the amount that is gained in years and students in a given amount of time.
>
> The ratio of students to the number of years.
>
> Since it is positive it increases.
>
> On a graph, for every point you move to the right on the x-axis. You move up 78 points on the y-axis.
>
> The slope in this equation means the students enrolled in 1996. Y = MX + B .
>
> The amount of students that enroll within a period of time.
>
> Every year the enrollment increases by 78 students.
>
> The slope here is 78 which means for each unit of time, (1 year) there are 78 more students enrolled.
>
> This is the rise in the number of students.
>
> The slope is the average amount of years it takes to get 156 more students enrolled in the school.
>
> Its how many times a year it increases.
>
> The slope is the increase of students per year.

Both groups had comparable ability to calculate the slope (several in both groups calculated ■x/■y). What is more valuable is the ability to understand what the slope means in context, whether that context arises in one of their other math courses or courses in other disciplines. Unless explicit attention is devoted to emphasizing the conceptual understanding of what the slope means, the majority of students are unable to create viable interpretations on their own. And, without that understanding, they likely can't apply the mathematics to realistic situations.

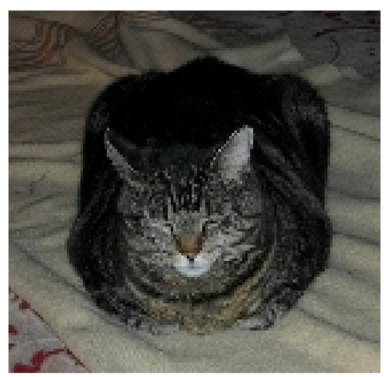
Many of us have heard complaints from colleagues in other disciplines about students seemingly not having learned key mathematical ideas, often the equation of a line. Too many math courses stress the manipulative technique for finding the equation without emphasizing the underlying conceptual understanding or realistic contexts in which such problems arise. In other disciplines, linear functions do not arise as: Find the equation of the line through (1,3) and (5,11). Instead, one has data on two quantities that follows a roughly linear pattern and has to find (and use) a line that fits the data. If students have as much difficulty understanding the slope, it is no wonder they can't connect what they learn in math classes to what they see in other courses.

Moreover, if students can't make their own connections with a concept as simple as the slope (which they undoubtedly encountered in previous math courses), they can't create meaningful interpretations and connections for more sophisticated concepts, such as the significance of the base (growth or decay factor) in an exponential function, the power in a power function, or the parameters in a sinusoidal model.

Based on our study, we cannot simply concentrate on teaching mathematical techniques. It is just as important to stress conceptual understanding and the meaning of the mathematics. This can and should be accomplished by using realistic examples and problems and by forcing the students to think, not just to manipulate symbols. If we don't, we won't adequately prepare them for successive mathematics courses, for courses in other disciplines, and for using mathematics on the job and throughout their lives.

[2] Cat Lapping

The New York Times published an article on November 12, 2010 describing how a cat "drinks." Because cats cannot close their mouths, they are unable to create a suction like humans can and therefore use their tongues to sip. High speed photography reveals that a cat laps 4 times a second and "drinks" about 3/100 teaspoons per lap. How long would it take a cat to lap up a cup of milk?



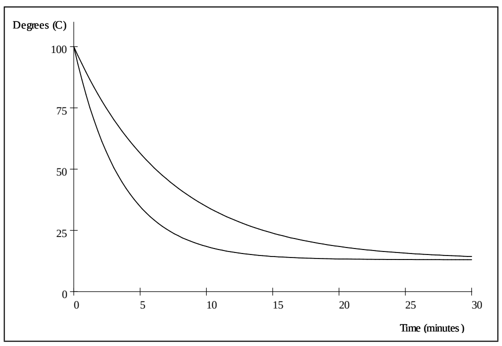
[3] Query

The product of two consecutive odd numbers is 399. What are the numbers?

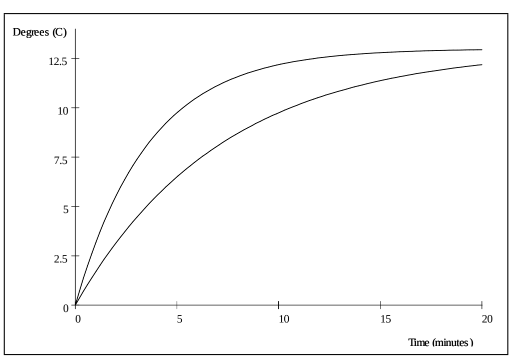
[4] Cooling and Warming

(These problems were suggested by Frank Wattenberg, U.S. Military Academy.)

a. Two cups of boiling tea were placed on separate counters in the same room. Next to one cup was a fan blowing air over the cup. Which of the two graphs below represent the temperature of the tea water in the cup next to the fan? (Time in minutes is measured along the horizontal axis, and degrees (C) is measured along the vertical axis.) Explain the reasoning for your choice.



b. Two cans of cold sodas were taken out of the refrigerator and placed on separate counters in the same room. Next to one soda was a fan blowing air over the soda can. Which of the two graphs below represent the temperature of the soda in the can next to the fan? (Time in minutes is measured along the horizontal axis, and degrees (C) is measured along the vertical axis.) Explain the reasoning for your choice.



[5] Notices

1. Jennifer Beecher is the McGraw-Hill Representative for Contemporary College Algebra 563.584.6323, [jennifer_beecher@mcgraw-hill.com]

2. Joint Mathematics Meetings, New Orleans, LA, January 6-9, 2011

3. **Reunion for Those Interested in Refocusing College Algebra**

   Joint Mathematics Meetings, New Orleans, LA

   Friday, January 7, 5:30 p.m. to 7:30 p.m., Grand Chateau, Sheraton Hotel

   Organizer: Don Small, US Military Academy, Sponsor: CRAFTY

4. A copy of the White Paper synthesizing the recommendations on Standards and Assessments in K-12 Mathematics from the CBMS Forum last October is posted on the CBMS website at http://www.cbmsweb.org/Forum 2/CBMS Forum White paper.pdf.

5. Deadline for contributions to the February Newsletter is February 1, 2011. Opinion articles, suggestions for writing assignments, small group in-class activities, small group out-of-class projects, Queries, announcements, etc. are welcomed.

6. To subscribe to this Newsletter, write to Don Small, Department of Mathematics, U.S. Military Academy, West Point, NY 10996 or contact him via e-mail at don-small@usma.edu.